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Dynamic Response of Plates due To Positive and Negative Phases of Localized Blast Load

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ABSTRACT: Localized blast loadings are often overlooked during the structural system design process despite their potential detrimental effects. These forces are unique and often generate irregular responses on structural slab constructions and may require certain attention. In this research, a set of slab models with semi-rigid boundary conditions on its sides and with variations in thickness are subjected to localized blast loading at certain locations. Localized blast loadings are modelled as blast load with linear function. The main system responses that are observed are the transversal deflections at midspan and the internal stresses of the system, particularly the maximum principle stress and minimum principle stress. Four loading phases are included in the analysis, namely: the positive phase, the zero phase, the negative phase, and the free vibration phase. Analyses are carried out utilizing a numeric approach termed the Modified Bolotin Method. Deflections resulting from various load positions on the set of slab model are then compared side-by-side. Stresses are calculated on all slab models with the localized blast loading applied at midspan and the results are presented as stress contours. Based on the results from this research, additional plate thickness can reduce plate deflection. The location of blast load also affect the value of plate deflection in the mid span. The maximum and minimum principle stress also reduced by the addition of plate thickness..

KEYWORDS: Localized blast load, Thickness, Modified Bolotin Method, principle stress.

I. INTRODUCTION

Over the last decades considerable attention has been raised on the behaviour of engineering structures under blast or impact loading. The use of explosives by terrorist groups around the world that target civilian buildings and other structures is becoming a growing problem in modern societies. Usually the casualties from such a detonation are not only related to instant fatalities as a consequence of the direct release of energy, but mainly to structural failures that might occur and could result in extensive life loss. Slab is one of the structural components that must designed against blast loading.

An explosion is characterised by the rapid release of energy as a mass of reactive material is converted into an extremely dense region of high pressure gas. This gas rapidly expands and displaces the surrounding air, causing a pressure disturbance a blast wave to propagate away from the centre of the explosive at supersonic speed [1]. Blast load consists of two phases which positive and negative phases. The positive phase starts from the beginning of the explosion which causes a collection of gases that have high pressure until the gas pressure returns to normal. After the blast load has passed its positive phase the load behaviour changes to negative pressure [2].

Several methods can be used to solve the dynamic response of the plates, these methods are the finite element method, the finite difference method, the Rayleigh-Ritz method and the Modified Bolotin Method. Modified Bolotin Method is a development of Bolotin Method which was first introduced in 1960. Bolotin Method can solve the plate dynamics problem by combining various types of plate edge supports [3]. Furthermore, the method was developed by Vijayakumar in 1971 through his article "A New Method for Analysis of Flexural Vibration of Rectangular Orthotropic Plates" [4]. In 2000, Modified Bolotin Method concept was developed by Pavner to solve dynamic plate problem with trigonometric function [5]. Modified Bolotin Method has the advantage in solving plate equations because it can get accurate solutions in the high mode.

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Equation of motion are obtain from classical plate theory based on Kirchhoff-Love Plate Theory or Thin Plate Theory [6]. This theory is extension from Euler-Bernoulli beam theory where the moment-curvature relationship is derived. Relationship moment, stiffness, strain and stress of a plate element with transverse load are derived based on Kirchhoff-Love Plate Theory and Theory of Elasticity.

The type of support used in modelling is very important because it will affect the plate response. A simply supported slab has often been the object of research because the analysis process is quite easier to solve. But the analysis of simply supported plate can't represent the real conditions [7]. In this work, the slab plate is modelled as thin plates, allowing the rotations and the vertical deformations along its edges.

II. LITERATURE REVIEW

Consider a rectangular plate of thickness h with a and b are dimension along the x - and y -axes, respectively. According to the classic theory of thin plates and dynamic of structures, the deflection of the concrete plate is governed by the following partial differential equation.

$$D_x \left(\frac{\partial^4 w(x,y,t)}{\partial x^4} \right) + 2B \left(\frac{\partial^4 w(x,y,t)}{\partial x^2 \partial y^2} \right) + D_y \left(\frac{\partial^4 w(x,y,t)}{\partial y^4} \right) + \gamma h \left(\frac{\partial w(x,y,t)}{\partial t} \right) + \rho h \left(\frac{\partial^2 w(x,y,t)}{\partial t^2} \right) = p_z(x,y,t) \quad (1)$$

Where $w(x,y,t)$ is the vertical deflection of the orthotropic plate at point (x,y) and time t , D_x is the flexural rigidity of plate in the x -direction, B is the torsional rigidity, D_y is the flexural rigidity of plate in the y -direction, ρ is the mass density of the plate, h is the thickness of plate, and γ is the critical damping ratio of the plate.

Figure 1 shows a localized blast loading on the structural surfaces. For the analysis purpose, the actual reduction of the initial pressure can be assumed as a triangular pressure impulse (dashed line on the graphic). The actual duration of the positive phase is replaced by a fictitious duration. The procedure to calculate the parameter of this load is based on Karlos and Solomos Technical reports [8].

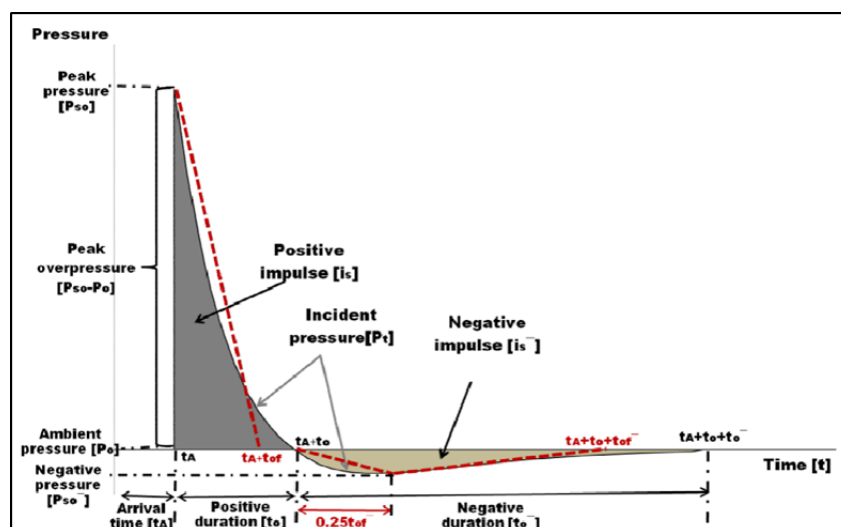


Fig. 1. Substitution of actual incident pressure curve by triangular pulses and definition of relevant fictitious times (Karlos, V and Solomos, G.)

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The function of linear blast loading, $P(t)$, can be expressed by the following equation:

$$p_s(t) = \begin{cases} 0 & t \leq t_{of} \\ -p_{so} \left(\frac{t - t_o}{0.25 t_{of}^-} \right) & t_o < t \leq t_o + 0.25 t_{of}^- \\ -p_{so} \left(1 - \frac{t - (t_o + 0.25 t_{of}^-)}{0.75 t_{of}^-} \right) & t_o + 0.25 t_{of}^- < t \leq t_o + t_{of}^- \end{cases} \quad (2)$$

Figure 2 shows a graphic to calculate the positive and negative phase parameters of blast loading.

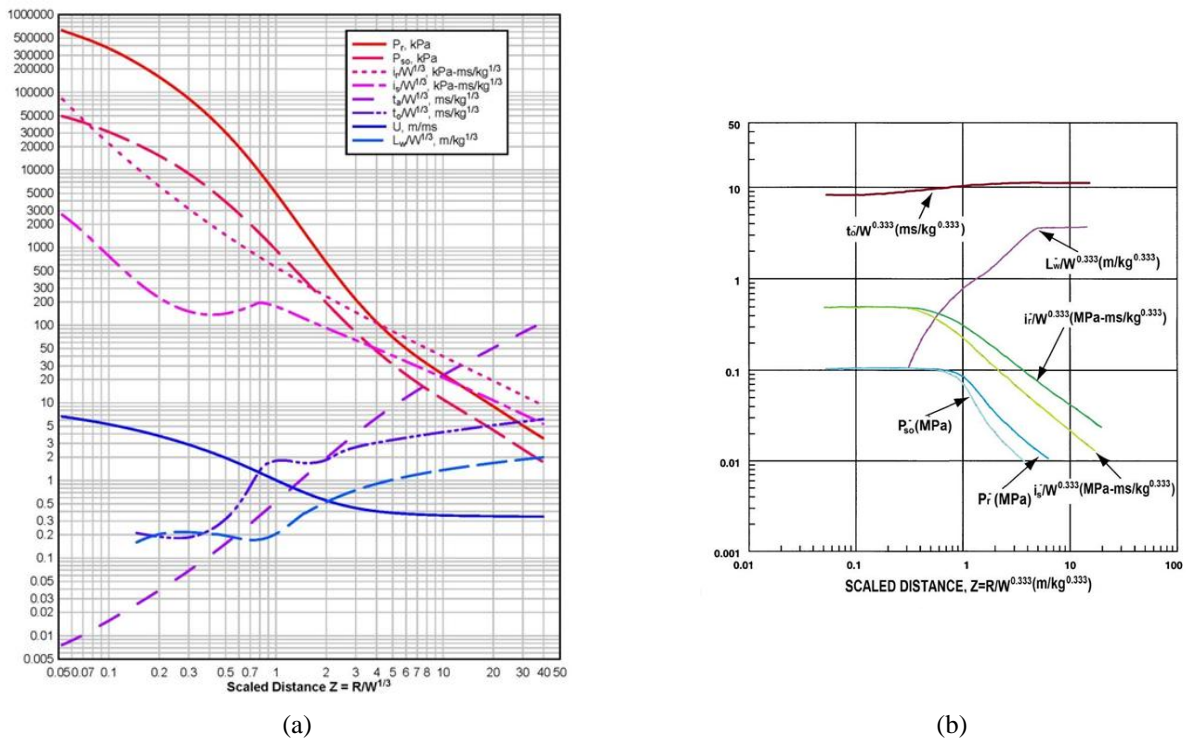


Fig. 2. Parameter of Blast Loading (a) Positive phase (b) Negative Phase (Karlou, V and Solomos, G.)

III. PLATE DYNAMIC RESPONSE

In the first part of this paper, the free vibration of plate is studied using Levy's solution. The free vibration solution of the system is set as

$$w(x, y, t) = W(x, y) T(t) = W(x, y) \sin \omega t \quad (3)$$

Where $W(x, y)$ is only a function of the position coordinates and ω is the circular frequency of the system. The Eigen frequencies are analogous to the case of a plate with simply supported at all edges, which can be expressed as

$$\omega_{mn}^2 = \frac{\pi^4}{\rho h} \left[D_x \left(\frac{p}{a} \right)^4 + 2B \left(\frac{pq}{ab} \right)^2 + D_y \left(\frac{q}{b} \right)^4 \right] \quad (4)$$

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Where p and q are real number to be solved from a system of two transcendental equation, obtained from the solution of two auxiliary Levy's problem, also known as the modified Bolotin method.

The solution of the free vibration problem in Eq. (4) can be obtained from

$$W(x, y) = X(x) \sin\left(\frac{q\pi y}{b}\right) \quad (5)$$

Which satisfied the elastic vertical translation and rotational boundary condition [9]. The solution of the characteristic equation become

$$X(x) = A_1 \cosh\left(\frac{\beta\pi x}{ab}\right) + A_2 \sinh\left(\frac{\beta\pi x}{ab}\right) + A_3 \cos\left(\frac{p\pi x}{a}\right) + A_4 \sin\left(\frac{p\pi x}{a}\right) \quad (6)$$

Where

$$\beta = \sqrt{\frac{2B}{D_x} a^2 q^2 + b^2 p^2}$$

Equation (6) gives the general form of the eigenmode of the plate in the x - direction. Substituting the eigenmode according to Eq.(6) into the boundary condition along $x=0$ and $x=a$, result in the linear algebra equation in terms of A_i , as follows:

$$[a] \{A\}^T = \{0\}^T \quad (7)$$

Where a_{ij} are the coefficient of $[a]$.

To obtain a non-trivial solution, it is necessary to propose that the determinant of Eq.(7) is zero. The determinant of Eq. (7) is called the transcendental equation of the first auxiliary Levy's problem. The second auxiliary Levy-type problem in the y -axis can be determined analogously to the above formulations.

The mode number p and q , respectively in the x - and y -directions of the system can be solved by solving the transcendental equation simultaneously. The solution cannot be determined analytically. Hence the Mathematica software is used to obtain solution numerically. The eigenfunction of the orthotropic plate is therefore given by

$$w(x, y) = \sum_{p=1}^{\infty} \sum_{q=1}^{\infty} X_{pq}(x) Y_{pq}(y) \quad (8)$$

The general solution in the time domain is given by

$$T_{pq}^*(t) = \int_0^t \left[\frac{P_z(x, y, t)}{\rho h Q_{pq}} \int_{x=0}^a X_{pq}(x) dx \int_{y=0}^b Y_{pq}(y) dy \left(\frac{e^{-\xi \omega_{pq}(t-\tau)}}{\omega_D} \sin(\omega_D(t-\tau)) \right) \right] d\tau \quad (9)$$

Where $p(x, y, t)$ is the decaying function dynamic blast loading, respectively, for positive phase and negative phase according to Eq.(2). Finally, the dynamic deflection of the plate is found by multiplying Eqs. (8) and (9).

IV. ANALYSIS

Numerical calculation are carried out for different parameters of the problem. The number of modes in the x - and y -direction is taken as $m=1,2,\dots,5$ and $n=1,2,\dots,5$ by considering the convergence of the eigenvalues. In this work, a finite rectangular plate is considered as shown in Fig.3. The size of the plate is 7x5 m. For comparison sake, three values of the plate thickness are considered $h=0.20m$, $h=0.25m$ and $h=0.3m$.

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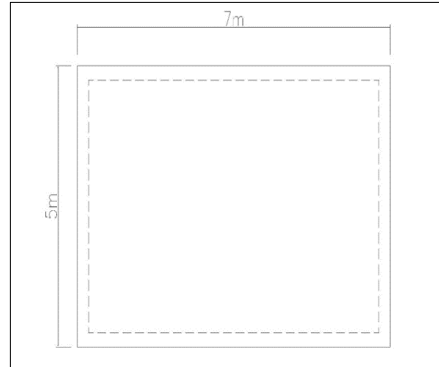


Fig.3. The geometry of plate

The physical characteristics of the plate are in table 1.

Table 1. Physical characteristic of plate

Plate Model		1 st Model	2 nd Model	3 rd Model
Notation	Parameter [unit]			
ρ	Mass density [kg/m^3]	2400	2400	2400
E_{cx}	Elastic Modulus x-direction [MPa]	25700	25700	25700
E_{cy}	Elastic Modulus y-direction [MPa]	23500	23500	23500
ν_x	Poisson Ratio x-direction	0.2	0.2	0.2
ν_y	Poisson Ratio x-direction	0.15	0.15	0.15
ξ	Damping ratio [%]	5	5	5
k_1	Boundary condition x- [N.m/rad]	$2,5 \times 10^8$	$2,5 \times 10^8$	$2,5 \times 10^8$
k_2	Boundary condition y- [N.m/rad]	$2,5 \times 10^8$	$2,5 \times 10^8$	$2,5 \times 10^8$

As shown in equation (2) blast load function consists of several discontinued functions, so in the analysis needs to be divided into several phases. The analysis divided in to: 1st Phase (Positive phase), 2nd Phase (Zero Phase), 3rd Phase (Negative phase), 4th Phase (Free Vibration phase). Figure 4 show analysis phase based on blast loading function.

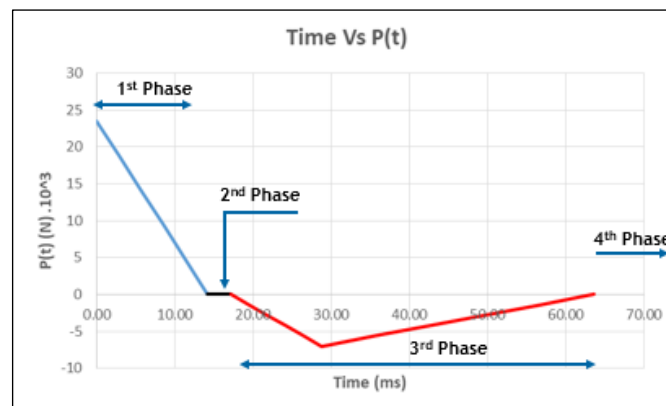


Fig.4. Analysis phase

The blast loading parameter which include in analysis shown in table 2.

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Table 2. Parameter of blast loading

Notation	value	unit	note
Z	6.03	m/kg ^(1/3)	Scaling laws (Assumption)
W	100	kg	Mass of explosive (Assumption)
P _{so}	23500	Newton [N]	Positive phase amplitude
P _{so} ⁻	7000	Newton [N]	Negative phase amplitude
t _a	0	Sec [s]	Initial time
t _{of}	0.0142	Sec [s]	Positive fictitious duration
t _o	0.0172	Sec [s]	Positive fictitious + zero phase duration
t _{of} ⁻	0.0464	Sec [s]	Negative fictitious duration

Three different locations of blast loading are studied in detail to determine the mid-point deflection of plate with the emphasis in the effect of the negative phase of the load. Figure 5 shown the location of blast loading in this work.

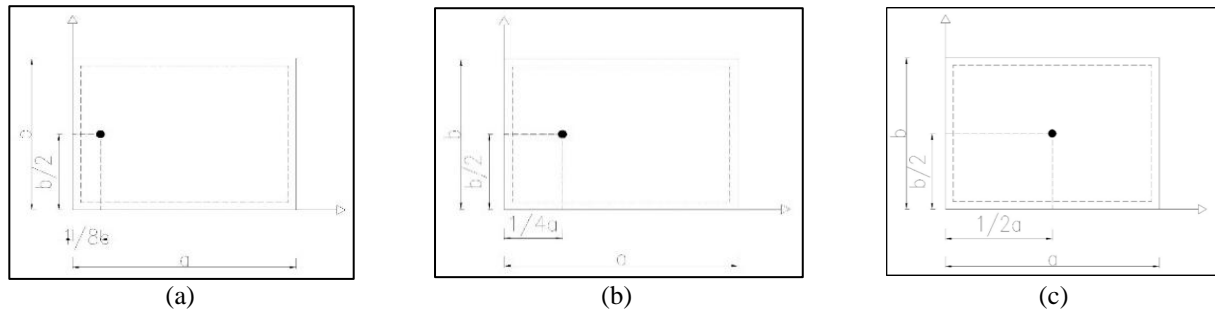


Fig. 5 Location of blast loading (a) 1/8a (b) 1/4a (c) 1/2a

V. NUMERICAL RESULTS

Using the procedure of the last section, some result have been obtained for a rectangular orthotropic plate subjected to a localized blast load. Table 3 shows the natural frequencies of the system for the first 5 mode in x and y direction. It can be seen from the table that the natural frequencies increases as the mode number and thickness of plate are increases.

Table 3. Computed natural frequencies of plate

Natural Frequency and Natural Period													
m	n	t = 20 cm				t = 23 cm				t = 25 cm			
		p		ω_n rad/s	T_n s	p		ω_n rad/s	T_n s	p		ω_n rad/s	T_n s
		q	q			q	q						
1	1	1.25	1.41	204	0.031	1.25	1.41	236	0.027	1.25	1.41	257	0.024
	2	1.15	2.47	494	0.013	1.15	2.48	569	0.011	1.15	2.48	620	0.010
	3	1.10	3.49	931	0.007	1.10	3.50	1073	0.006	1.10	3.50	1169	0.005
	4	1.08	4.50	1513	0.004	1.08	4.51	1745	0.004	1.08	4.51	1901	0.003
	5	1.06	5.51	2241	0.003	1.06	5.52	2584	0.002	1.06	5.52	2816	0.002
2	1	2.39	1.28	338	0.019	2.39	1.28	390	0.016	2.39	1.28	424	0.015
	2	2.26	2.40	616	0.010	2.26	2.41	710	0.009	2.26	2.41	773	0.008
	3	2.19	3.45	1049	0.006	2.19	3.46	1209	0.005	2.19	3.46	1317	0.005
	4	2.15	4.48	1630	0.004	2.15	4.48	1880	0.003	2.15	4.49	2047	0.003
	5	2.12	5.49	2358	0.003	2.12	5.50	2719	0.002	2.12	5.51	2962	0.002

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3	1	3.44	1.20	562	0.011	3.44	1.20	647	0.010	3.44	1.20	703	0.009
	2	3.34	2.33	825	0.008	3.34	2.33	950	0.007	3.34	2.34	1034	0.006
	3	3.27	3.40	1251	0.005	3.27	3.41	1441	0.004	3.27	3.41	1569	0.004
	4	3.21	4.44	1828	0.003	3.21	4.45	2107	0.003	3.21	4.45	2295	0.003
	5	3.18	5.47	2554	0.002	3.18	5.47	2944	0.002	3.18	5.48	3207	0.002
4	1	4.46	1.15	867	0.007	4.46	1.16	998	0.006	4.46	1.16	1085	0.006
	2	4.39	2.27	1120	0.006	4.39	2.28	1290	0.005	4.39	2.28	1403	0.004
	3	4.32	3.35	1537	0.004	4.32	3.36	1770	0.004	4.32	3.36	1927	0.003
	4	4.27	4.40	2109	0.003	4.27	4.41	2430	0.003	4.27	4.42	2646	0.002
	5	4.23	5.44	2831	0.002	4.23	5.44	3263	0.002	4.23	5.45	3554	0.002
5	1	4.39	2.27	1120	0.006	4.39	2.28	1290	0.005	4.39	2.28	1403	0.004
	2	5.42	2.23	1498	0.004	5.42	2.23	1724	0.004	5.42	2.24	1875	0.003
	3	5.36	3.31	1906	0.003	5.36	3.32	2195	0.003	6.39	3.28	2953	0.002
	4	5.31	4.37	2472	0.003	5.31	4.37	2847	0.002	5.31	4.38	3099	0.002
	5	5.27	5.41	3189	0.002	5.27	5.41	3675	0.002	5.27	5.42	4002	0.002

Figure 6 shows the time history of the mid-point deflection of the plate subjected to blast loading for different locations of loading. The result shows that the dynamic deflection at mid-point decrease as the position of the load closes to the edge of the plate. The dynamic deflection at mid-point also decrease as the increase of plate thickness.

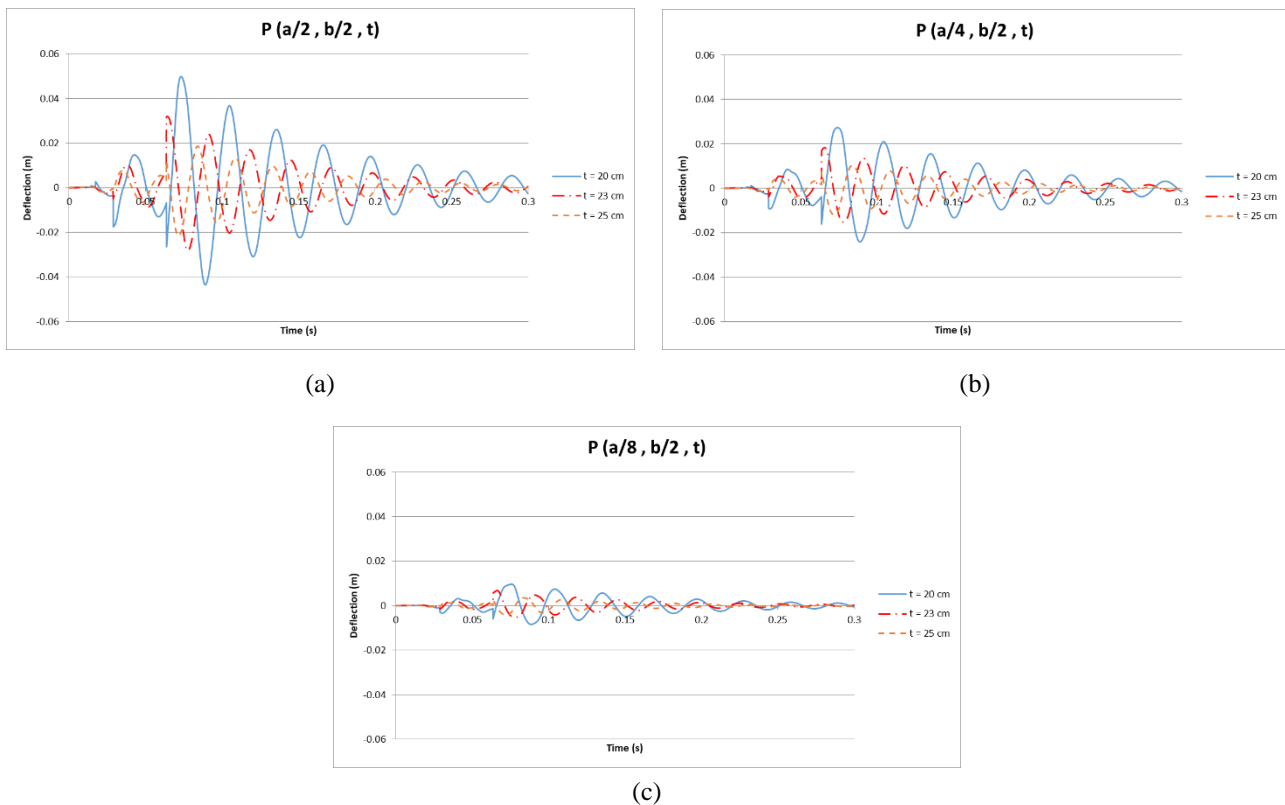


Fig.6 Dynamic deflection time history of plate subjected to blast load (a) blast load at 1/2a (b) blast load at 1/4a (c) blast load at 1/8a

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Figure 7 shows the maximum principal stress contour of plate with 20cm thickness. The maximum value of principle stress does not occurs in the mip-point of plate even though the load is located in the mid-point. The maximum principle stress contour of 23cm and 25cm plate are similar with 20cm plate. The maximum value of principle stress for 20cm, 23cm and 25cm plate, respectively 50MPa, 25MPa and 20MPa.

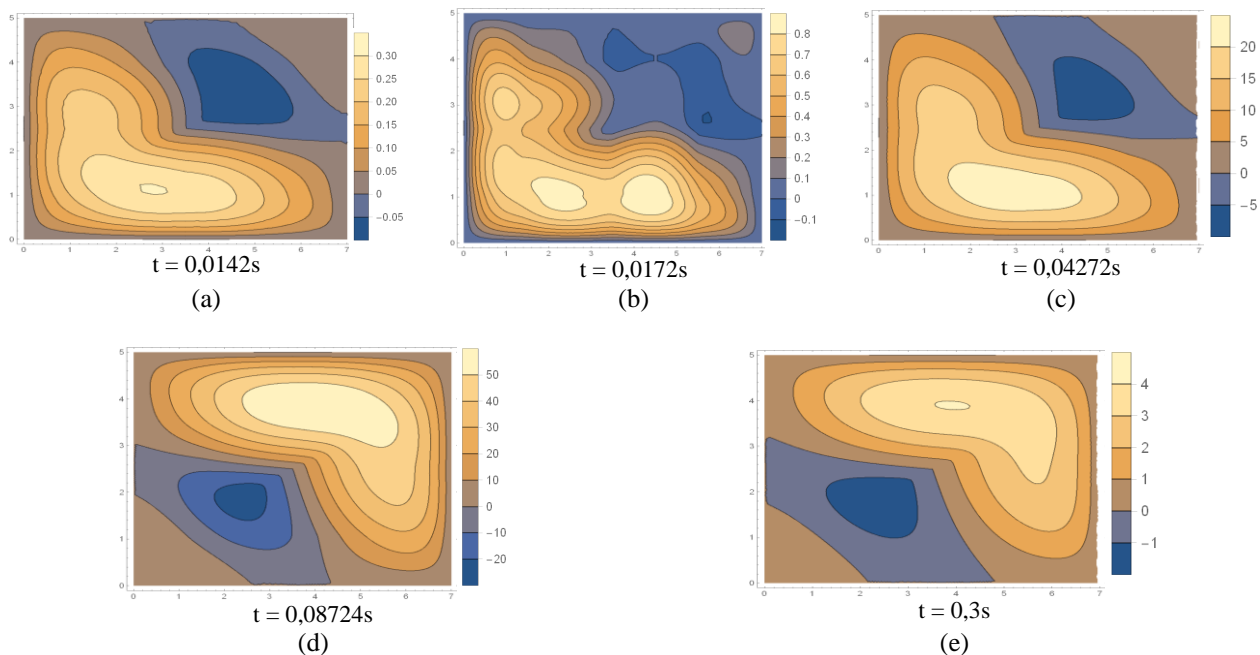
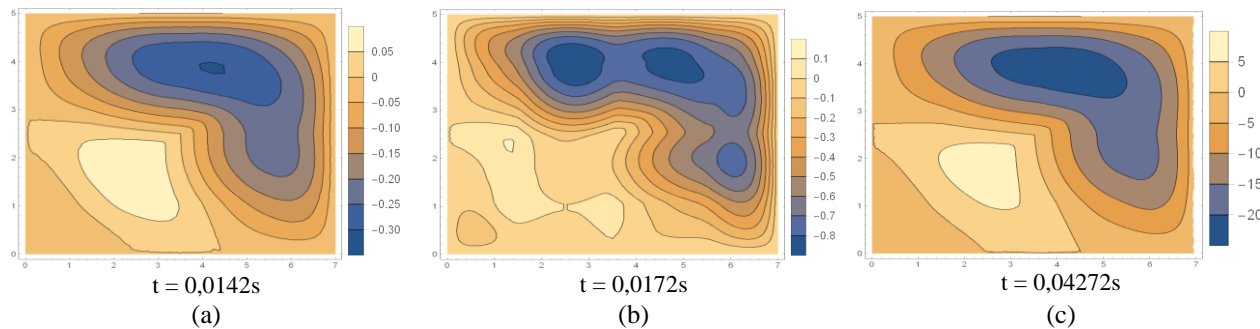


Figure 7. Maximum principal stress contour of plate (a) $t = 0.0142s$ (b) $t = 0.0172s$ (c) $t = 0.04272s$ (d) $t = 0.0872s$ (e) $t = 0.3s$

Figure 8 shows the minimum principal stress contour of plate with 20cm thickness. The minimum value of principle stress does not occurs in the mip-point of plate even though the load is located in the mip-point. The minimum principle stress contour of 23cm and 25cm plate are similar with 20cm plate. The minimum value of principle stresses for 20cm, 23cm and 25cm plate, are 50MPa, 25MPa and 20MPa.



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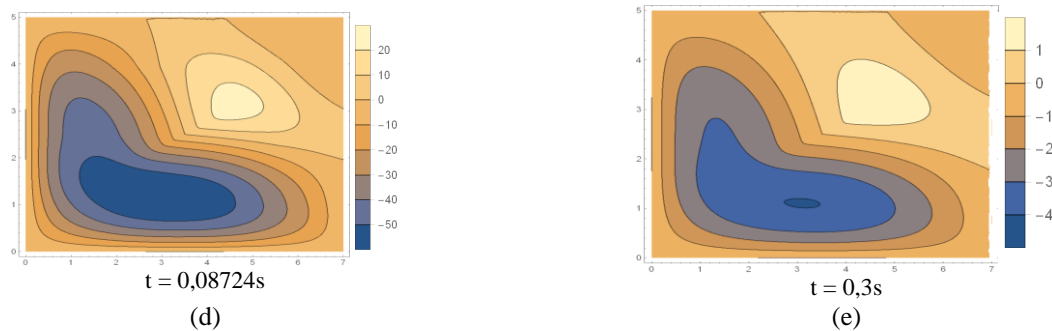


Figure 8. Minimum principal stress contour of plate (a) $t = 0.0142s$ (b) $t = 0.0172s$ (c) $t = 0.04272s$ (d) $t = 0.0872s$ (e) $t = 0.3s$

VI. CONCLUSION

Based on the above study, the following conclusions can be drawn.

1. Natural frequency of plate with thickness 20cm, 23cm and 25cm are 204 rad/s, 236 rad/s and 257 rad/s. It means the addition of plate thickness causes the plate's natural frequency getting higher so the plate becomes more rigid.
2. The largest plate deflection occurs after the negative phase occurs. This phenomena occurs in every case. Even though the load is no longer working on the plate, the initial condition that occurs due to the end of the previous phase's loading duration can actually cause a deflection value greater than the deflection during loading phase.
3. The addition of plate thickness by 3cm reduces the maximum absolute deflection by 35.86%, while the addition of plate thickness by 2cm reduces the maximum absolute deflection by 32.20%.
4. Shifting the load as far as 1/4 span reduces the maximum absolute deflection by 45.32% while moving the load as far as 3/4 span reduces the maximum absolute deflection by 64.41%.
5. The addition of plate thickness by 3cm can reduce the maximum and minimum principal stress values by 60%, while the addition of plate thickness by 2cm does not reduce the maximum and minimum principal stress values.

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