

Properties of N^* -closed sets and $*N$ -closed Sets in Topology

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ABSTRACT: The purpose of this paper is to define and study a new classes of sets, called the class of N^* -closed sets, which lies between the class of α -closed sets and the class of strongly αg -closed sets and $*N$ -closed sets in topology. Also, the basic properties of these N^* -closed sets, N^* -open sets, $N^*T_{1/2}$ -spaces, N^* -continuous functions, N^* -irresolute functions and N^* -connected spaces, $*N$ -closed functions, pre- $*N$ -closed functions and always $*N$ -closed functions are studied in this paper.

KEYWORDS: α -open sets, αg -closed sets, gp -closed sets and g^*p -closed sets, $g\alpha$ -closed sets and strongly αg -closed sets, preopen sets, α -closed sets, gp -closed sets and αg -open sets, gp -continuity, pre- gp -continuity, pre- gp -closed functions.

I. INTRODUCTION

After, introduction of g -closed sets in topology for the first time by Levine [15], it is noticed that many of the weaker forms of closed sets have been generalized in topological spaces. In 1963, 1965, 1982 and 1986, respectively, N. Levine [14], O. Njastad [25], A. S. Mashhour et al [17] and D. Andrijevic [1] have defined and studied the concepts of semiopen sets, α -open sets, preopen sets and semipreopen sets in topology. In 1994, 1995, 1998, 2000, 2002, 2006, respectively, H. Maki et al [16], Dontchev [9], Noiri et al [26], M. K. R. S. Veera Kumar [29] & [30], P. Rajrajeswari et al [27], have defined and studied the concepts of $g\alpha$ -closed sets, αg -closed sets, gsp -closed sets, gp -closed sets, g^* -closed sets, g^*p -closed sets and strongly αg -closed sets. Recently, Navalagi [20 & 23 & 21] has defined and studied the new classes of generalized closed sets, called G^* -closed sets, g^*sp -closed sets and $*G$ -closed sets in topology. The purpose of this paper is to define and study a new class of sets, called N^* -closed sets and the class of $*N$ -closed sets in topological spaces. Also, we define and study their basic properties.

II. PRELIMINARIES

Throughout this paper (X, τ) and (Y, σ) (or simply X and Y) always means topological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a subset of space X . We denote the closure of A and the interior of A by $Cl(A)$ and $Int(A)$ respectively. A subset A of a space X is called regular open (in brief, r -open) if $A = Int Cl(A)$ and regular closed (in brief, r -closed) if $A = Cl Int(A)$.

The following definitions and results are useful in the sequel :

Definition 2.1 : A subset A of a space X is said to be:

- (i) preopen [17] if $A \subset Int Cl(A)$.
- (ii) semiopen [14] if $A \subset Cl Int(A)$.
- (iii) semipreopen [1] if $A \subset Cl Int Cl(A)$.
- (iv) α -open [25] if $A \subset Int Cl Int(A)$.

The complement of a preopen (resp. semiopen, semipreopen, α -open) set of a space X is called preclosed [10] (resp. semiclosed [4], semipreclosed [1], α -closed [18]).

Definition 2.2 [2]: The union of all semipreopen sets contained in A is called the semipreinterior of A and is denoted by $spInt(A)$.

$pInt(A)$ [19] and $sInt(A)$ [8], $\alpha Int(A)$ [18] can be similarly defined.

Definition 2.3 [2]: The intersection of all semipreclosed sets containing A is called the semipreclosure of A and is denoted by $spCl(A)$.

$pCl(A)$ [10] and $sCl(A)$ [7], $\alpha Cl(A)$ [18] can be similarly defined.

DEFINITION 2.4 : A subset A of a space X is called :

- (i) generalized closed set (in brief , g -closed) set [15] if $Cl(A) \subset U$ whenever $A \subset U$ and U is open in X .
- (ii) generalized semiclosed (in brief , gs -closed) set [2] if $sCl(A) \subset U$ whenever $A \subset U$ and U is open in X .
- (iii) generalized semipreclosed (in brief , gsp -closed) set [9] if $spCl(A) \subset U$ whenever $A \subset U$ and U is open in X .
- (iv) generalized preclosed (in brief , gp -closed) set [26] if $pCl(A) \subset U$ whenever $A \subset U$ and U is open in X .
- (v) gpr -closed set [11] if $pCl(A) \subset U$ whenever $A \subset U$ and U is r -open in X .
- (vi) g^* -closed set [29] if $Cl(A) \subset U$ whenever $A \subset U$ and U is g -open in X .
- (vii) g^* -preclosed (in brief , g^*p -closed) set [30] if $pCl(A) \subset U$ whenever $A \subset U$ and U is g -open in X .
- (viii) g^*sp -closed set [20] if $spCl(A) \subset U$ whenever $A \subset U$ and U is g -open in X .
- (ix) gr^* -closed set [12] if $rCl(A) \subset U$ whenever $A \subset U$ and U is g -open in X .
- (x) α -generalized closed (in brief , αg -closed) set [16] if $\alpha Cl(A) \subset U$ whenever $A \subset U$ and U is open in X .
- (xi) G^* -closed set [21] if $gspCl(A) \subset U$ whenever $A \subset U$ and U is g -open set in X .
- (xii) strongly αg -closed [27] if $\alpha Cl(A) \subset U$ whenever $A \subset U$ and U is g -open set in X .
- (xiii) αg^* -closed [28] if $\alpha Cl(A) \subset U$ whenever $A \subset U$ and U is αg -open set in X .
- (xiv) gp^* -closed [13] if $Cl(A) \subset U$ whenever $A \subset U$ and U is gp -open set in X .
- (xv) αg^*p -closed [28] if $pCl(A) \subset U$ whenever $A \subset U$ and U is αg -open set in X .
- (xvi) $*G$ -closed [23] if $gsCl(A) \subset U$ whenever $A \subset U$ and U is g -open set in X .
- (xvii) $g\#$ - closed [31] if $Cl(A) \subset U$ whenever $A \subset U$ and U is g -open set in X .

Definition 2.5 [22]: The union of all gsp -open sets contained in A is called the gsp -interior of A and is denoted by $gspInt(A)$.

Definition 2.6 [22]: The intersection of all gsp -closed sets containing A is called the gsp -closure of A and is denoted by $gspCl(A)$.

Lemma 2.7 [22] : For every subset $A \subset X$, the following hold.

- (i) $gspCl(X-A) = X-gspInt(A)$.
- (ii) $gspInt(X-A) = X-gspCl(A)$.

Definition 2.8 : A function $f : X \rightarrow Y$ is called :

- (i) precontinuous [17] if the inverse image of each open set of Y is preopen in X .
- (ii) α -continuous [18] if the inverse image of each open set of Y is α -open in X .

Definition 2.9 : A function $f : X \rightarrow Y$ is called :

- (i) g -continuous [3] if the inverse image of each closed set of Y is g -closed in X .
- (ii) $st\alpha g$ -continuous [27] if the inverse image of each closed set of Y is $st\alpha g$ -closed in X .



- (iii) g^* -continuous [29] if the inverse image of each closed set of Y is g^* -closed in X .
- (iv) αg^* -continuous [28] if the inverse image of each closed set of Y is αg^* -closed in X .
- (v) gp^* -continuous [13] if the inverse image of each closed set of Y is gp^* -closed in X .
- (vi) $g\#$ -continuous [31] if the inverse image of each closed set of Y is $g\#$ -closed in X .
- (vii) αg^*p -continuous [28] if the inverse image of each closed set of Y is αg^*p -closed in X .

Definition 2.10 [6]: A space X is said to be g -Hausdroff if whenever x and y are distinct points of X there exist disjoint g -open sets U and V such that $x \in U$ and $y \in V$.

Definition 2.11[21]: A space X is said to be G^* -Hausdroff if whenever x and y are distinct points of X there exist disjoint G^* -open sets U and V such that $x \in U$ and $y \in V$.

III. PROPERTIES OF N^* -CLOSED SETS

We, define the following.

Definition 3.1: The union of all αg -open sets contained in A is called the αg -interior of A and is denoted by $\alpha gInt(A)$.

Definition 3.2: The intersection of all αg -closed sets containing A is called the αg -closure of A and is denoted by $\alpha gCl(A)$.

We, prove the following.

Lemma 3.3: For every subset $A \subset X$, the following hold.

- (i) $\alpha gCl(X-A) = X - \alpha gInt(A)$.
- (ii) $\alpha gInt(X-A) = X - \alpha gCl(A)$.

Next, we introduce the following.

Definition 3.4: A subset A of a space X is said to be N^* -closed set if $\alpha gCl(A) \subset U$ whenever $A \subset U$ and U is g -open in X .

The complement of a N^* -closed set is called N^* -open set.

In view of Definitions-2.4 and 3.4, we have the following implications hold true:

Lemma 3.5: Every, closed (and hence α -closed, strongly αg -closed, αg^* -closed, g^* -closed, gp^* -closed) set is N^* -closed set. Hence,

Lemma 3.6: Every, open (and hence α -open, strongly αg -open, αg^* -open, g^* -open, gp^* -open) set is N^* -open set.

Lemma 3.7: Every, N^* -closed set is G^* -closed set.

Lemma 3.8: N^* -closed and gp -closed sets are independent.

We, define the following.

Definition 3.9: A subset A of a space X is said to be N^{**} -closed set if $\alpha gCl(A) \subset U$ whenever $A \subset U$ and U is open in X .

The complement of a N^{**} -closed set is called N^{**} -open set.

Lemma 3.10: Every, closed (and hence α -closed, strongly αg -closed, N^* -closed, g^* -closed) set is N^{**} -closed set.

Lemma 3.11: A subset A of space X is called N^* -open set if $F \subset \alpha gInt(A)$ whenever $F \subset A$ and F is g -closed set in X

We, define the following



Definition 3.12 : The union of all N^* -open sets which contained in A is called the N^* -interior of A and is denoted by $N^*Int(A)$.

Definition 3.13 : The intersection of all N^* -closed sets containing set A is called the N^* -closure of A and is denoted $N^*Cl(A)$.

Lemma 3. 14 : Let $x \in X$, then $x \in N^*Cl(A)$ if and only if $V \cap A \neq \emptyset$ for every N^* -open set V containing x .

Remark 3.15 : The N^* -closure of a set A is not always N^* -closed set.

We , state the following.

Lemma 3.16: Let A and B be subsets of X . Then

(i) $N^*Cl(\emptyset) = \emptyset$ and $N^*Cl(X) = X$

(ii) If $A \subset B$, $N^*Cl(A) \subset N^*Cl(B)$.

(iii) $A \subset N^*Cl(A)$

Lemma 3.17: Let A and B be subsets of X . Then

(i) $N^*Cl(A) = N^*Cl(N^*Cl(A))$

(ii) $N^*Cl(A) \cup N^*Cl(B) \subset N^*Cl(A \cup B)$

(iii) $N^*Cl(A \cap B) \subset N^*Cl(A) \cap N^*Cl(B)$

Theorem3.18: If $\alpha gInt(A) \subset B \subset A$ and A is N^* -open, then B is N^* -open.

Proof: Obvious.

We, define the following.

Definition 3.19: A function $f: X \rightarrow Y$ is called N^* -continuous if $f^{-1}(V)$ is N^* -closed in X for every closed subset V of Y .

Clearly, every continuous (and hence α -continuous , $\sigma \alpha g$ -continuous , αg^* -continuous , g^* -continuous and gp^* -continuous) function is N^* -continuous.

Definition 3.20 : A function $f: X \rightarrow Y$ is called N^* -irresolute continuous if $f^{-1}(V)$ is N^* -closed in X for every N^* - closed subset V of Y .

Clearly , every N^* -irresolute function is N^* - continuous.

Definition 3.21 : A function $f : X \rightarrow Y$ is called strongly N^* -continuous if the inverse image of each N^* -open set of Y is open in X .

Clearly , every strongly N^* - continuous function is N^* -irresolute .

Definition 3.22 : A function $f : X \rightarrow Y$ is called (α , N^*) -continuous if the inverse image of each α -open set of Y is N^* -open in X .

Definition 3.23 : A function $f : X \rightarrow Y$ is called (g^* , N^*) -continuous if the inverse image of each g^* -open set of Y is N^* -open in X .

Definition 3.24: A function $f : X \rightarrow Y$ is called $(\alpha g^* , N^*)$ -continuous if the inverse image of each αg^* -open set of Y is N^* -open in X .

Definition 3.25: A function $f : X \rightarrow Y$ is called (gp^* , N^*) -continuous if the inverse image of each gp^* -open set of Y is N^* -open in X .

We, prove the following

Theorem 3.26 : Let $f : X \rightarrow Y$ be a function, then the following are equivalent :

(i) f is N^* -continuous.

(ii) The inverse image of each open set A in Y is N^* -open in X .

Proof : Easy.

Theorem 3.27 : Let $f : X \rightarrow Y$ be N^* -continuous and $g : Y \rightarrow Z$ be continuous , then $g \circ f : X \rightarrow Z$ is N^* -continuous.

Proof : Let B be an open set in Z , then $g^{-1}(B)$ is an open set in Y . Again, f is N^* -continuous function and $g^{-1}(B)$ is open set in Y , then $f^{-1}(g^{-1}(B)) = (gof)^{-1}(B)$ is N^* -open set in X . Thus, gof is N^* -continuous function.

Theorem 3.28 : A function $f : X \rightarrow Y$ is N^* -irresolute if for each N^* -open set A of Y , $f^{-1}(A)$ is N^* -open set in X .

Proof : Easy.

Theorem 3.29 : If $f : X \rightarrow Y$ is N^* -irresolute function, then for each $B \subset X$, $f(N^*Cl(A)) \subset N^*Cl(f(A))$.

Proof : Let $B \subset X$, then $N^*Cl(f(B))$ is N^* -closed set in Y , but f is N^* -irresolute function, therefore $f^{-1}(N^*Cl(f(B)))$ is N^* -closed set in X . Again, as $B \subset f^{-1}(f(B)) \subset f^{-1}(N^*Cl(f(B)))$. Thus, $N^*Cl(B) \subset N^*Cl(f^{-1}(N^*Cl(f(B)))) = f^{-1}(N^*Cl(f(B)))$. This implies that $f(N^*Cl(B)) \subset N^*Cl(f(B))$.

We, state the following.

Theorem 3.30 : Let $f : X \rightarrow Y$ be N^* -irresolute and $g : Y \rightarrow Z$ be (α, N^*) -continuous, then $gof : X \rightarrow Z$ is (α, N^*) -continuous.

Theorem 3.31 : Let $f : X \rightarrow Y$ be N^* -irresolute and $g : Y \rightarrow Z$ be (g^*, N^*) -continuous, then $gof : X \rightarrow Z$ is (g^*, N^*) -continuous.

Theorem 3.32 : Let $f : X \rightarrow Y$ be N^* -irresolute and $g : Y \rightarrow Z$ be $(\alpha g^*, N^*)$ -continuous, then $gof : X \rightarrow Z$ is $(\alpha g^*, N^*)$ -continuous.

Theorem 3.33 : Let $f : X \rightarrow Y$ be N^* -irresolute and $g : Y \rightarrow Z$ be (gp^*, N^*) -continuous, then $gof : X \rightarrow Z$ is (gp^*, N^*) -continuous.

We, prove the following.

Theorem 3.34 : Let $f : X \rightarrow Y$ be (α, N^*) -continuous and $g : Y \rightarrow Z$ be α -continuous, then $gof : X \rightarrow Z$ is N^* -continuous.

Proof : Let V be an open set in Z , then $g^{-1}(V)$ is an α -open set in Y since g is α -continuous. Again, f is (α, N^*) -continuous and $g^{-1}(V)$ is an α -open set in Y , then $(gof)^{-1}(V)$ is N^* -open set in X . Therefore, gof is N^* -continuous.

Theorem 3.35 : Let $f : X \rightarrow Y$ be (g^*, N^*) -continuous and $g : Y \rightarrow Z$ be g^* -continuous, then $gof : X \rightarrow Z$ is N^* -continuous.

Proof : Let V be an open set in Z , then $g^{-1}(V)$ is an g^* -open set in Y since g is g^* -continuous. Again, f is (g^*, N^*) -continuous and $g^{-1}(V)$ is an g^* -open set in Y , then $(gof)^{-1}(V)$ is N^* -open set in X . Therefore, gof is N^* -continuous.

Theorem 3.36 : Let $f : X \rightarrow Y$ be $(\alpha g^*, N^*)$ -continuous and $g : Y \rightarrow Z$ be αg^* -continuous, then $gof : X \rightarrow Z$ is N^* -continuous.

Proof : Let V be an open set in Z , then $g^{-1}(V)$ is an αg^* -open set in Y since g is αg^* -continuous. Again, f is $(\alpha g^*, N^*)$ -continuous and $g^{-1}(V)$ is an αg^* -open set in Y , then $(gof)^{-1}(V)$ is N^* -open set in X . Therefore, gof is N^* -continuous.

Theorem 3.37 : Let $f : X \rightarrow Y$ be (gp^*, N^*) -continuous and $g : Y \rightarrow Z$ be gp^* -continuous, then $gof : X \rightarrow Z$ is N^* -continuous.

Proof : Let V be an open set in Z , then $g^{-1}(V)$ is an gp^* -open set in Y since g is gp^* -continuous. Again, f is (gp^*, N^*) -continuous and $g^{-1}(V)$ is an gp^* -open set in Y , then $(gof)^{-1}(V)$ is N^* -open set in X . Therefore, gof is N^* -continuous.

We, define the following.

Definition 3.38 : A subset A of a space X is called N^* -clopen if it is both N^* -open set and N^* -closed set.

Clearly, every clopen set is N^* -clopen set, every N^* -clopen set is N^* -open and N^* -clopen set is N^* -closed set also.

Definition 3.39 : A function $f : X \rightarrow Y$ is said to be totally N^* -continuous if the inverse image of each open set of Y is N^* -clopen set in X .

Clearly, every totally continuous function is totally N^* -continuous.

We, define the following.

Definition 3.40 : A space X is said to be N^* -Hausdorff if for any pair of distinct points x, y of X , there exist disjoint N^* -open sets A and B such that $x \in A$ and $y \in B$.



Next , we prove the following.

Theorem 3.41 : If $f : X \rightarrow Y$ is totally N^* -continuous injection and Y is T_0 -space , then X is N^* - Hausdorff space.

Proof : Let x and y be any two distinct points of X . Then , $f(x) \neq f(y)$ since f is injective. As Y is T_0 -space , there exists an open set V containing say, $f(x)$ but not $f(y)$. Then $x \in f^{-1}(V)$ and $y \notin f^{-1}(V)$. Since f is totally N^* -continuous , $f^{-1}(V)$ is N^* -clopen set of X . Also, $x \in f^{-1}(V)$ and $y \in X - f^{-1}(V)$. Therefore , X is N^* - Hausdorff space.

We , define the following.

Definition 3.42 : A space X is said to be N^* -connected if X cannot be expressed as the union of two non-empty disjoint N^* -open sets .

The routine proof of the following is omitted.

Theorem 3.43 : If f is a totally N^* -continuous function from a N^* -connected space X onto any space Y , then Y is an indiscrete space.

IV. BASIC PROPERTIES OF N^* -CLOSED SETS

We, define the following.

Definition 4.1 : The union of all gp-open sets contained in A is called the gp-interior of A and is denoted by $gpInt(A)$. $gInt(A)$ [5] can be similarly defined .

Definition 4.2: The intersection of all gp-closed sets containing A is called the gp-closure of A and is denoted by $gpCl(A)$. $gCl(A)$ [5] can be similarly defined .

Note 4.3: For a subset A of a space X , the following hold true :

(i) every closed set is g -closed set & every g -closed set is gp-closed set .Hence we have ,

$gCl(A) \subset Cl(A)$ and $gspCl(A) \subset gCl(A)$, therefore we obtain , $gspCl(A) \subset gCl(A) \subset Cl(A)$.

(ii) every , closed set is α -closed ; α -closed set is preclosed ; preclosed set is gp-closed ;

gp-closed set is gsp -closed . Hence we have , $\alpha Cl(A) \subset Cl(A)$; $pCl(A) \subset \alpha Cl(A)$;

$gpCl(A) \subset pCl(A)$; $gspCl(A) \subset gpCl(A)$, therefore we obtain , $gspCl(A) \subset gpCl(A) \subset pCl(A) \subset \alpha Cl(A) \subset Cl(A)$.

(iii) every , α -closed set is αg -closed ; αg -closed set is gp-closed ; gp-closed set

is gsp -closed . Hence we have , $\alpha gCl(A) \subset \alpha Cl(A)$; $gpCl(A) \subset \alpha gCl(A)$; $gspCl(A) \subset gpCl(A)$; therefore we obtain , $gspCl(A) \subset gpCl(A) \subset \alpha gCl(A) \subset \alpha Cl(A) \subset Cl(A)$.

Note 4.4 : For a subset A of a space X , the following hold true :

(i) A is closed set if $A = Cl(A)$,

(ii) A is preclosed set if $A = pCl(A)$,

(iii) A is semipreclosed set if $A = spCl(A)$,

(iv) A is gsp -closed set if $A = gspCl(A)$,

(v) A is gp-closed set if $A = gpCl(A)$,

(vi) A is αg -closed set if $A = \alpha gCl(A)$.

We , define the following .



Definition 4.5 : A subset A of a space X is called *N-closed set if $gpCl(A) \subset U$ whenever $A \subset U$ and U is g-open in X.

In view of Definitions- 2.4 & 4.5 , we have the following implications :

Lemma 4.6: Every closed (and hence preclosed , g-closed) set is *N-closed set.

Lemma 4.7: (i) Every $g^\#$ -closed set is *N-closed set .

Pf. Let A be a $g^\#$ -closed set in X.Let U be a g-open set such that $A \subset U$. Since every g-open set is α g-open set and hence we have $Cl(A) \subset U$.But , $gpCl(A) \subset Cl(A) \subset U$.Thus , A is *N-closed set.

We state the following .

Lemma 4.8 : (i) Every *N-closed set is G^* -closed set,(ii) Every N^* -closed set is *N-closed set,(iv)Every N^* -closed set is G^* -closed set,(v)Every g^* -closed set is *N-closed set,(vi) Every g^*p -closed set is *N-closed set,(vii) *N-closed and gpr -closed sets are independent,(viii) Every gr^* -closed set is *N-closed set, (ix) gp -closed (and hence gsp -closed) sets and *N-closed sets are independent each other and (x) Every αg^*p -closed set is *N-closed set .

We, prove the following .

Lemma 4.9 : Let A be a *N-closed set of space X.Then,

- (i) $gpCl(A)-A$ does not contain any nonempty g-closed set.
- (ii) If $A \subset B \subset gpCl(A)$, then B is also a *N-closed set of X.

Proof . (i) Let F be a g-closed set contained in $gpCl(A) - A$, then $gpCl(A) \subset X - F$ since $X - F$ is g-open with $A \subset X - F$ and A is *N-closed set. Then, $F \subset (X-gpCl(A)) \cap (gpCl(A)-A) \subset (X-gpCl(A)) \cap gpCl(A) = \emptyset$. Thus , $F = \emptyset$.
 (ii)Let G be a g-open set of X such that $B \subset G$. Then , $A \subset G$. Since $A \subset G$ and A is *N-closed set , then $gpCl(A) \subset G$. Then, $gpCl(B) \subset gpCl(gpCl(A)) = gpCl(A)$ since $B \subset gpCl(A)$. Thus , $gpCl(B) \subset gpCl(A) \subset G$. Hence , B is also a *N-closed set of X.

Next, we introduce the following .

Definition 4.10 : A space X is said to be a :

- (i) $*T_n$ - space if every *N-closed set in it is closed .
- (ii) $**T_p$ -space if every *N-closed set in it is preclosed .
- (iii) $**T_g$ - space if every *N-closed set in it is g-closed.
- (iv) $\#T_g$ -space if every *N-closed set in it is $g^\#$ -closed.
- (v) $**T_N$ -space if every N^* -closed set in it is αg^*p -closed.
- (vi) $*T_{NG}$ -space if every *G-closed set in it is *N-closed .

Lemma 4.11: A subset A of space X is called *N-open set if $F \subset gpInt(A)$ whenever $F \subset A$ and F is g-closed set in X

We , define the following

Definition 4.12 : The union of all *N-open sets which contained in A is called the *N-interior of A and is denoted by $*NInt(A)$.

Definition 4.13 : The intersection of all *N-closed sets containing set A is called the *N-closure of A and is denoted $*NCl(A)$.

Lemma 4.14 : Let $x \in X$, then $x \in *NCl(A)$ if and only if $V \cap A \neq \emptyset$ for every *N-open set V containing x.

Remark 4.15 : The *N-closure of a set A is not always *N-closed set.



We , state the following.

Lemma 4.16: Let A and B be subsets of X. Then

- (i) $*NCl(\emptyset) = \emptyset$ and $*NCl(X) = X$
- (ii) If $A \subset B$, $*NCl(A) \subset *NCl(B)$.
- (iii) $A \subset *NCl(A)$

Lemma 4.17: Let A and B be subsets of X. Then

- (i) $*NCl(A) = *NCl(*GCl(A))$
- (ii) $*NCl(A) \cup *NCl(B) \subset *NCl(A \cup B)$
- (iii) $*NCl(A \cap B) \subset *NCl(A) \cap *NCl(B)$

Theorem 4.18: If $gpInt(A) \subset B \subset A$ and A is *N-open, then B is *N-open.

Proof: Obvious.

We, define the following.

Definition 4.19 : A function $f: X \rightarrow Y$ is called *N -continuous if $f^{-1}(V)$ is *N-closed in X for every closed subset V of Y.

Clearly, every continuous (and hence precontinuous , g-continuous , αg^*p -continuous , g^* -continuous and $g\#$ -continuous) function is *N-continuous.

Definition 4.20 : A function $f: X \rightarrow Y$ is called *N-irresolute continuous if $f^{-1}(V)$ is *N-closed in X for every *N- closed subset V of Y.

Clearly , every *N-irresolute function is *N- continuous

Lemma 4.21 : Let $f: X \rightarrow Y$ be a *N-continuous function .

- (i) If X is a $*T_n$ - space , then f is continuous.
- (ii) If X is a $**T_p$ -space , then f is precontinuous.
- (iii) If X is a $**T_g$ - space , then f is g-continuous.
- (iv) If X is a $\#T_g$ -space , then f is $g\#$ -continuous.
- (v) If X is a $**T_N$ -space , then f is αg^*p -continuous.

Lemma 4.22 : Let $f: X \rightarrow Y$ be a G^* -continuous function .If X is $*T_{NG}$ -space , then f is *N-continuous .

We, define the following.

Definition 4.23 : A function $f: X \rightarrow Y$ is called strongly *N-continuous if the inverse image of each *N-open set of Y is open in X.

Definition 4.24 : A function $f: X \rightarrow Y$ is called $(g, *N)$ -continuous if the inverse image of each g-open set of Y is *N-open in X.

Definition 4.25 : A function $f: X \rightarrow Y$ is called $(p, *N)$ -continuous if the inverse image of each preopen set of Y is *N-open in X.

Definition 4.26: A function $f: X \rightarrow Y$ is called $(g^*, *N)$ -continuous if the inverse image of each preopen set of Y is *N-open in X.

Definition 4.27 : A function $f: X \rightarrow Y$ is called $(g^*p, *N)$ -continuous if the inverse image of each g^*p -open set of Y is *N-open in X.

Definition 4.28 : A function $f: X \rightarrow Y$ is called $(G^*, *N)$ -continuous if the inverse image of each G^* -open set of Y is *N-open in X.

We, define the following.

Definition 4.29: A space X is said to be $*N$ -Hausdorff if whenever x and y are distinct points of X , there exist disjoint $*N$ -open sets U and V such that $x \in U$ and $y \in V$.

Definition 4.30: A space X is said to be g^*p -Hausdorff if whenever x and y are distinct points of X there exist disjoint g^*p -open sets U and V such that $x \in U$ and $y \in V$.

Definition 4.31: A space X is said to be g^* -Hausdorff if whenever x and y are distinct points of X there exist disjoint g^* -open sets U and V such that $x \in U$ and $y \in V$.

Definition 4.32: A space X is said to be p -Hausdorff if whenever x and y are distinct points of X there exist disjoint preopen sets U and V such that $x \in U$ and $y \in V$.

Next, we prove the following.

Theorem 4.33: Let X be a space and Y be Hausdorff. If $f: X \rightarrow Y$ be $*N$ -continuous injective, then X is $*N$ -Hausdorff.

Proof: Let x and y be any two distinct points of X . Then $f(x)$ and $f(y)$ are distinct points of Y , because f is injective. Since Y is Hausdorff, there exist two disjoint open sets U and V such that $f(x) \in U$ and $f(y) \in V$. Since f is $*N$ -continuous and $U \cap V = \emptyset$, we have $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint $*N$ -open sets in X such that $x \in f^{-1}(U)$ and $y \in f^{-1}(V)$. Hence X is $*N$ -Hausdorff space.

Theorem 4.34: Let X be a space and Y be $*N$ -Hausdorff. If $f: X \rightarrow Y$ be $*N$ -irresolute injective, then X is $*N$ -Hausdorff.

Proof: Let x and y be any two distinct points of X . Then $f(x)$ and $f(y)$ are distinct points of Y , because f is injective. Since Y is $*N$ -Hausdorff, there exist two disjoint $*N$ -open sets U and V such that $f(x) \in U$ and $f(y) \in V$. Since f is $*N$ -irresolute and $U \cap V = \emptyset$, we have $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint $*N$ -open sets in X such that $x \in f^{-1}(U)$ and $y \in f^{-1}(V)$. Hence X is $*N$ -Hausdorff space.

Theorem 4.35: Let X be a space and Y be g -Hausdorff. If $f: X \rightarrow Y$ be $(g, *N)$ -continuous injective, then X is $*N$ -Hausdorff.

Proof: Let x and y be any two distinct points of X . Then $f(x)$ and $f(y)$ are distinct points of Y , because f is injective. Since Y is g -Hausdorff, there exist two disjoint g -open sets U and V such that $f(x) \in U$ and $f(y) \in V$. Since f is $(g, *N)$ -continuous and $U \cap V = \emptyset$, we have $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint $*N$ -open sets in X such that $x \in f^{-1}(U)$ and $y \in f^{-1}(V)$. Hence X is $*N$ -Hausdorff space.

Theorem 4.36: Let X be a space and Y be g^*p -Hausdorff. If $f: X \rightarrow Y$ be $(g^*p, *N)$ -continuous injective, then X is $*N$ -Hausdorff.

Proof: Let x and y be any two distinct points of X . Then $f(x)$ and $f(y)$ are distinct points of Y , because f is injective. Since Y is g^*p -Hausdorff, there exist two disjoint g^*p -open sets U and V such that $f(x) \in U$ and $f(y) \in V$. Since f is $(g^*p, *N)$ -continuous and $U \cap V = \emptyset$, we have $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint g^*sp -open sets in X such that $x \in f^{-1}(U)$ and $y \in f^{-1}(V)$. Hence X is g^*sp -Hausdorff space.

Theorem 4.37: Let X be a space and Y be $*N$ -Hausdorff. If $f: X \rightarrow Y$ be strongly $*N$ -continuous injective, then X is Hausdorff.

Proof: Let x and y be any two distinct points of X . Then $f(x)$ and $f(y)$ are distinct points of Y , because f is injective. Since Y is $*N$ -Hausdorff, there exist two disjoint $*N$ -open sets U and V such that $f(x) \in U$ and $f(y) \in V$. Since f is strongly- $*N$ -continuous and $U \cap V = \emptyset$, we have $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint open sets in X such that $x \in f^{-1}(U)$ and $y \in f^{-1}(V)$. Hence X is Hausdorff space.

We, state the following.

Theorem 4.38: Let X be a space and Y be p -Hausdorff. If $f: X \rightarrow Y$ be $(p, *N)$ -continuous injective, then X is $*N$ -Hausdorff.

Theorem 4.39: Let X be a space and Y be g^* -Hausdorff. If $f: X \rightarrow Y$ be $(g^*, *N)$ -continuous injective, then X is $*N$ -Hausdorff.

Theorem 4.40: Let X be a space and Y be G^* -Hausdorff. If $f: X \rightarrow Y$ be $(G^*, *N)$ -continuous injective, then X is $*N$ -Hausdorff.

We define the following.

Definition 4.41: A function $f: X \rightarrow Y$ is said to be $*N$ -open if the image of each open set of X is $*N$ -open in Y .

Definition 4.42: A function $f:X \rightarrow Y$ is said to be always $\ast N$ -open if the image of each $\ast N$ -open set of X is $\ast N$ -open in Y .

Clearly, every always- $\ast N$ -open function is $\ast N$ -open.

Definition 4.43 : A function $f:X \rightarrow Y$ is said to be $\ast N$ -closed if the image of each closed set of X is $\ast N$ -closed in Y .

Definition 4.44 : A function $f:X \rightarrow Y$ is said to be pre- $\ast N$ -closed if the image of each preclosed set of X is $\ast N$ -closed in Y .

Definition 4.45 : A function $f:X \rightarrow Y$ is said to be always- $\ast N$ -closed if the image of each $\ast N$ -closed set of X is $\ast N$ -closed in Y .

Clearly, every always- $\ast N$ -closed function is $\ast N$ -closed.

We, state the following.

Theorem 4.46: Let $f:X \rightarrow Y$ be a bijection. Then the following are equivalent.

- (i) f is always- $\ast N$ -open.
- (ii) f is always- $\ast N$ -closed.
- (iii) f^{-1} is $\ast N$ -irresolute.

We, define the following.

Definition 4.47: A function $f:X \rightarrow Y$ is said to be strongly $\ast N$ -open if the image of each $\ast N$ -open set of X is open in Y .

We, state the following.

Theorem 4.48: Let $f:X \rightarrow Y$ and $g:Y \rightarrow Z$ be any two functions. Then,

- (i) If f is $\ast N$ -open and g is always $\ast N$ -open, then $g \circ f$ is $\ast N$ -open.
- (ii) If f is strongly $\ast N$ -open and g is $\ast N$ -open, then $g \circ f$ is always $\ast N$ -open.
- (iii) If f is always $\ast N$ -open and g is strongly $\ast N$ -open, then $g \circ f$ is strongly $\ast N$ -open.

We, define the following.

Definition 4.49: A function $f:X \rightarrow Y$ is said to be strongly $\ast N$ -closed if the image of each $\ast N$ -closed set of X is closed in Y .

We, state the following.

Theorem 4.50: Let $f:X \rightarrow Y$ and $g:Y \rightarrow Z$ be any two functions. Then,

- (i) If f is $\ast N$ -closed and g is always $\ast N$ -closed, then $g \circ f$ is $\ast N$ -closed.
- (ii) If f is strongly $\ast N$ -closed and g is $\ast N$ -closed, then $g \circ f$ is always $\ast N$ -closed.
- (iii) If f is always $\ast N$ -closed and g is strongly $\ast N$ -closed, then $g \circ f$ is strongly $\ast N$ -

closed.

Definition 4.51: A bijection $f:X \rightarrow Y$ is called $\ast N$ -homeomorphism if f is both $\ast N$ -continuous function and $\ast N$ -open function.

Theorem 4.52 : Every homeomorphism is an $\ast N$ -homeomorphism..

We, define the following.

Definition 4.53 : A bijection $f:X \rightarrow Y$ is said to be always $\ast N$ -homeomorphism if both f and f^{-1} are $\ast N$ -irresolute functions.

Theorem 4.54: Every always $\ast N$ -homeomorphism is an $\ast N$ -homeomorphism.

Proof: Obvious.



V. CONCLUSION

We have defined and studied two classes of generalized closed sets, namely, N^* -closed sets and $*N$ -closed sets in topological spaces in this paper. In the upcoming research papers, our aim is to establish some more basic properties of these new generalized closed sets in topology.

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