

Properties of always *G-Continuous Functions in Topological Spaces

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ABSTRACT: The purpose of this paper is to define and study a new class of sets, called the class of *G-closed sets in topological spaces. Also, we define and study concepts of *G-continuous functions, always *G-continuous functions, *G-open functions and *G-closed functions in this paper.

KEYWORDS : semiopen sets, semiclosed sets, gs-closed sets and gs-open sets, gs-continuity, G*-continuity, pre-gs-continuity, pre-gs-closed functions.

I. INTRODUCTION

Levine [14] generalized the closed set to generalized closed set (g-closed set) in topology for the first time. Since then it is noticed that some of the weaker forms of closed sets have been generalized. In 1963 and 1986, respectively, N. Levine [13] and D. Andrijevic [1] have defined and studied the concepts of semiopen sets and semipreopen sets in topology. In 1990 and 1995, respectively, Arya et al [2] Dontchev [10], have defined and studied the concepts of gs-closed sets and gsp-closed sets in topological spaces. Recently, Navalagi et al [17&20] and Navalagi [18] have defined and studied the new classes of generalized closed sets, called g*sp-closed sets and G*-closed in topology. The purpose of this paper is to define and study a new class of sets, called the class of *G-closed sets in topological spaces. Also, we define and study concepts of *G-continuous functions, always *G-continuous functions, *G-open functions and *G-closed functions in this paper.

II. PRELIMINARIES

Throughout this paper (X, τ) and (Y, σ) (or simply X and Y) always means topological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a subset of space X . We denote the closure of A and the interior of A by $Cl(A)$ and $Int(A)$, respectively. A subset A of a space X is called regular open if $A = Int Cl(A)$ and regular closed if $A = Cl Int(A)$.

The following definitions and results are useful in the sequel :

Definition 2.1 : A subset A of a space X is said to be:

- (i) semiopen [13] if $A \subset Cl Int(A)$.
- (ii) semipreopen [1] if $A \subset Cl Int Cl(A)$.

The complement of a semiopen (resp. semipreopen) set of a space X is called semiclosed [5] (resp. semipreclosed [1]) set in X .

Definition 2.2 : The union of all semipreopen sets contained in A is called the semipreinterior of A and is denoted by $spInt(A)$ [1].

$rInt(A)$ [23], $\alpha Int(A)$ [16] and $sInt(A)$ [9] can be similarly defined.

Definition 2.3: The intersection of all semipreclosed sets containing A is called the semipreclosure of A and is denoted by $spCl(A)$ [1].

$rCl(A)$ [23], $\alpha Cl(A)$ [16] and $sCl(A)$ [8] can be similarly defined.

DEFINITION 2.4 : A subset A of a space X is called :

- (i) generalized closed set (in brief, g-closed) set [14] if $Cl(A) \subset U$ whenever $A \subset U$ and U is open in X .
- (ii) generalized semiclosed (in brief, gs-closed) set [2] if $sCl(A) \subset U$ whenever $A \subset U$ and U is open in X .



- (iii) generalized semipreclosed (in brief, gsp-closed) set [10] if $spCl(A) \subset U$ whenever $A \subset U$ and U is open in X .
 - (iv) α -generalized closed (in brief, α g-closed) set [15] if $\alpha Cl(A) \subset U$ whenever $A \subset U$ and U is open in X .
 - (v) g^* -closed set [25] if $Cl(A) \subset U$ whenever $A \subset U$ and U is g -open in X .
 - (vi) $(gs)^*$ -closed set [11] if $Cl(A) \subset U$ whenever $A \subset U$ and U is gs -open in X .
 - (vii) G^* sp-closed set [17,20] if $spCl(A) \subset U$ whenever $A \subset U$ and U is g -open set in X .
 - (viii) G^* -closed set [18] if $gspCl(A) \subset U$ whenever $A \subset U$ and U is g -open set in X .
 - (ix) g^* s-closed in the sense of Navalagi [21] (it is denoted by $(N)g^*$ s-closed) set if $sCl(A) \subset U$ whenever $A \subset U$ and U is gs -open set in X .
 - (x) g^* s-closed in the sense of Anitha [24] (it is denoted by $(A)g^*$ s-closed) set if $sCl(A) \subset U$ whenever $A \subset U$ and U is g -open set in X .
 - (xi) gr -closed set [4] if $rCl(A) \subset U$ whenever $A \subset U$ and U is open in X .
 - (xii) gr^* -closed set [12] if $rCl(A) \subset U$ whenever $A \subset U$ and U is g -open in X .
 - (xiii) sg -closed set [3] if $sCl(A) \subset U$ whenever $A \subset U$ and U is semiopen in X
- The complement of a g -closed (resp. gs -closed, gsp -closed, α g-closed, g^* -closed, $(gs)^*$ -closed, pre-semiclosed, G^* -closed, $g^\#$ s-closed, $(N)g^*$ s-closed, $(A)g^*$ s-closed, gr -closed, gr^* -closed and sg -closed) set of X is called g -open (resp. gs -open, gsp -open, α g-open, g^* -open, $(gs)^*$ -open, pre-semi open, G^* -open, $g^\#$ s-open, $(N)g^*$ s-open, $(A)g^*$ s-open, gr -open, gr^* -open and sg -open) set of X .

Definition 2.5 [] : A space X is said to be g -Hausdroff if whenever x and y are distinct points of X there exist disjoint g -open sets U and V such that $x \in U$ and $y \in V$.

III. PROPERTIES OF ALWAYS *G-CONTINUOUS FUNCTIONS

We, recall the following.

Definition 3.1 : The union of all g -open sets contained in A is called the g -interior of A and is denoted by $gInt(A)$ [22].

$gInt(A)$ [6], $sgInt(A)$ [22] and $gspInt(A)$ [] can be similarly defined.

Definition 3.2: The intersection of all g -closed sets containing A is called the g -closure of A and is denoted by $gCl(A)$ [22].

$gCl(A)$ [6], $sgCl(A)$ [22] and $gspCl(A)$ [19] can be similarly defined.

Note 3.3: For a subset A of a space X , the following hold true :

- (i) every closed set is g -closed set & every g -closed set is gs -closed set .Hence we have , $gCl(A) \subset Cl(A)$ and $gsCl(A) \subset gCl(A)$, therefore we obtain , $gsCl(A) \subset gCl(A) \subset Cl(A)$.
- (ii) every , closed set is α -closed ; α -closed set is semiclosed ; semiclosed set is sg -closed ; sg -closed set is gs -closed . Hence we have , $\alpha Cl(A) \subset Cl(A)$; $sCl(A) \subset \alpha Cl(A)$; $sgCl(A) \subset sCl(A)$; $gsCl(A) \subset sgCl(A)$, therefore we obtain , $gsCl(A) \subset sgCl(A) \subset sCl(A) \subset \alpha Cl(A) \subset Cl(A)$.
- (iii) every , semiclosed set is semipreclosed ; semipreclosed set is gsp -closed ; sg -closed set is gsp -closed ; sg -closed set is gs -closed & gs -closed set is gsp -closed . Hence we have , $spCl(A) \subset sCl(A)$; $gspCl(A) \subset spCl(A)$; $gspCl(A) \subset sgCl(A)$; $gsCl(A) \subset sgCl(A)$; $gspCl(A) \subset gsCl(A)$; therefore we obtain , $gspCl(A) \subset gsCl(A) \subset sgCl(A) \subset sCl(A) \subset Cl(A)$.
- (iv) every , semipreclosed set is gsp -closed , α -closed set is α g-closed & α g-closed set is gsp -closed . Hence we have , $gspCl(A) \subset spCl(A)$; $\alpha gCl(A) \subset \alpha Cl(A)$ & $gspCl(A) \subset \alpha gCl(A)$; therefore we obtain , $gspCl(A) \subset \alpha gCl(A) \subset \alpha Cl(A) \subset Cl(A)$.

Note 3.4 : For a subset A of a space X , the following hold true :

- (i) A is closed set if $A = Cl(A)$,
- (ii) A is semiclosed set if $A = sCl(A)$,
- (iii) A is semipreclosed set if $A = spCl(A)$,
- (iv) A is gs -closed set if $A = gsCl(A)$,
- (v) A is sg -closed set if $A = sgCl(A)$,
- (vi) A is gsp -closed set if $A = gspCl(A)$.

We , define the following .

Definition 3.5 : A subset A of a space X is called $*G$ -closed set if $gCl(A) \subset U$ whenever $A \subset U$ and U is g -open in X .



In view of Definitions- 2.4 & 3.5 , we have the following implications :

Lemma 3.6 : Every closed (and hence , semiclosed & g -closed) set is $*G$ -closed set.

Lemma 3.7 : Every gr^* -closed (and hence , (N) g^*s -closed ; (A) g^*s -closed ; $g^\#s$ -closed ; g^* -closed ; (gs) * -closed) set is $*G$ -closed set.

Lemma 3.8 : Every $*G$ -closed set is gs -closed set (and hence G^* -closed set) .

Lemma 3.9 : $*G$ -closed sets and gsp -closed (and hence presemiclosed) sets are independent each other .

We, prove the following .

Lemma 3.10 : Let A be a $*G$ -closed set of space X. Then,

- (i) $gsCl(A)-A$ does not contain any nonempty g -closed set.
- (ii) If $A \subset B \subset gsCl(A)$, then B is also a $*G$ -closed set of X.

Proof . (i) Let F be a g -closed set contained in $gsCl(A) - A$, then $gsCl(A) \subset X - F$ since $X - F$ is g -open with $A \subset X - F$ and A is $*G$ -closed set. Then, $F \subset (X - gsCl(A)) \cap (gsCl(A) - A) \subset (X - gsCl(A)) \cap gsCl(A) = \emptyset$. Thus , $F = \emptyset$.

(ii) Let G be a g -open set of X such that $B \subset G$. Then , $A \subset G$. Since $A \subset G$ and A is $*G$ -closed set , then $gsCl(A) \subset G$. Then, $gsCl(B) \subset gsCl(gsCl(A)) = gsCl(A)$ since $B \subset gsCl(A)$. Thus , $gsCl(B) \subset gsCl(A) \subset G$. Hence , B is also a $*G$ -closed set of X.

Next, we introduce the following .

Definition 3.11 : A space X is said to be a :

- (i) $*T_c$ - space if every $*G$ -closed set in it is closed .
- (ii) $*T_{sc}$ -space if every $*G$ -closed set in it is semiclosed .
- (iii) $*T_g$ - space if every $*G$ -closed set in it is g -closed.
- (iv) $*T_G$ -space if every gs -closed set in it is $*G$ -closed.
- (v) $**T_G$ -space if every G^* -closed set in it is $*G$ -closed.
- (vi) $*T_{gs}$ -space if every $*G$ -closed set in it is (gs) * -closed .

Lemma 3.12: A subset A of space X is called $*G$ -open set if $F \subset gsInt(A)$ whenever $F \subset A$ and F is g -closed set in X

We , define the following

Definition 3.13 : The union of all $*G$ -open sets which contained in A is called the $*G$ -interior of A and is denoted by $*GInt(A)$.

Definition 3.14 : The intersection of all $*G$ -closed sets containing set A is called the $*G$ -closure of A and is denoted $*GCl(A)$.

Lemma 3. 15: Let $x \in X$, then $x \in *GCl(A)$ if and only if $V \cap A \neq \emptyset$ for every $*G$ -open set V containing x.

Remark 3.16: The $*G$ -closure of a set A is not always $*G$ -closed set.

We , state the following.

Lemma 3.17: Let A and B be subsets of X. Then

- (i) $*GCl(\emptyset) = \emptyset$ and $*GCl(X) = X$
- (ii) If $A \subset B$, $*GCl(A) \subset *GCl(B)$.
- (iii) $A \subset *GCl(A)$

Lemma 3.18: Let A and B be subsets of X. Then

- (i) $*GCl(A) = *GCl(*GCl(A))$
- (ii) $*GCl(A) \cup *GCl(B) \subset *GCl(A \cup B)$
- (iii) $*GCl(A \cap B) \subset *GCl(A) \cap *GCl(B)$

Theorem 3.19: If $gsInt(A) \subset B \subset A$ and A is $*G$ -open, then B is $*G$ -open.

Proof: Obvious.

We, define the following.

Definition 3.20 : A function $f: X \rightarrow Y$ is called $*G$ -continuous if $f^{-1}(V)$ is $*G$ -closed in X for every closed subset V of Y.

Definition 3.21 : A function $f: X \rightarrow Y$ is called always $*G$ - continuous if $f^{-1}(V)$ is $*G$ -closed in X for every $*G$ - closed subset V of Y.

Definition 3.22 : A function $f : X \rightarrow Y$ is called strongly $*G$ -continuous if the inverse image of each $*G$ -open set of Y is open in X .

Definition 3.23: A function $f : X \rightarrow Y$ is called $(g, *G)$ -continuous if the inverse image of each g -open set of Y is $*G$ -open in X .

Definition 3.24 : A function $f : X \rightarrow Y$ is called $(s, *G)$ -continuous if the inverse image of each semiopen set of Y is $*G$ -open in X .

Definition 3.25 : A function $f : X \rightarrow Y$ is called $(G^*, *G)$ -continuous if the inverse image of each semiopen set of Y is $*G$ -open in X .

Definition 3.26: A space X is said to be $*G$ -Hausdorff if whenever x and y are distinct points of X , there exist disjoint $*G$ -open sets U and V such that $x \in U$ and $y \in V$.

Definition 3.27: A space X is said to be s -Hausdorff if whenever x and y are distinct points of X there exist disjoint semi-open sets U and V such that $x \in U$ and $y \in V$.

Definition 28: A space X is said to be G^* -Hausdorff if whenever x and y are distinct points of X there exist disjoint g -open sets U and V such that $x \in U$ and $y \in V$.

Next, we prove the following.

Theorem 3.29: Let X be a space and Y be Hausdorff. If $f: X \rightarrow Y$ be $*G$ -continuous injective, then X is $*G$ -Hausdorff.

Proof : Let x and y be any two distinct points of X . Then $f(x)$ and $f(y)$ are distinct points of Y , because f is injective. Since Y is Hausdorff, there exist two disjoint open sets U and V such that $f(x) \in U$ and $f(y) \in V$. Since f is $*G$ -continuous and $U \cap V = \emptyset$, we have $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint $*G$ -open sets in X such that $x \in f^{-1}(U)$ and $y \in f^{-1}(V)$. Hence X is $*G$ -Hausdorff space.

Theorem 3.30 : Let X be a space and Y be $*G$ -Hausdorff. If $f: X \rightarrow Y$ be always $*G$ -continuous injective, then X is $*G$ -Hausdorff.

Proof : Let x and y be any two distinct points of X . Then $f(x)$ and $f(y)$ are distinct points of Y , because f is injective. Since Y is $*G$ -Hausdorff, there exist two disjoint $*G$ -open sets U and V such that $f(x) \in U$ and $f(y) \in V$. Since f is always $*G$ -continuous and $U \cap V = \emptyset$, we have $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint $*G$ -open sets in X such that $x \in f^{-1}(U)$ and $y \in f^{-1}(V)$. Hence X is $*G$ -Hausdorff space.

Theorem 3.31 : Let X be a space and Y be g -Hausdorff. If $f: X \rightarrow Y$ be $(g, *G)$ -continuous injective, then X is $*G$ -Hausdorff.

Proof : Let x and y be any two distinct points of X . Then $f(x)$ and $f(y)$ are distinct points of Y , because f is injective. Since Y is g -Hausdorff, there exist two disjoint g -open sets U and V such that $f(x) \in U$ and $f(y) \in V$. Since f is $(g, *G)$ -continuous and $U \cap V = \emptyset$, we have $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint $*G$ -open sets in X such that $x \in f^{-1}(U)$ and $y \in f^{-1}(V)$. Hence X is $*G$ -Hausdorff space.

Theorem 3.32: Let X be a space and Y be G^* -Hausdorff. If $f: X \rightarrow Y$ be $(G^*, *G)$ -continuous injective, then X is $*G$ -Hausdorff.

Proof : Let x and y be any two distinct points of X . Then $f(x)$ and $f(y)$ are distinct points of Y , because f is injective. Since Y is G^* -Hausdorff, there exist two disjoint G^* -open sets U and V such that $f(x) \in U$ and $f(y) \in V$. Since f is $(G^*, *G)$ -continuous and $U \cap V = \emptyset$, we have $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint $*G$ -open sets in X such that $x \in f^{-1}(U)$ and $y \in f^{-1}(V)$. Hence X is $*G$ -Hausdorff space.

Theorem 3.33 : Let X be a space and Y be $*G$ -Hausdorff. If $f: X \rightarrow Y$ be strongly $*G$ -continuous injective, then X is Hausdorff.

Proof : Let x and y be any two distinct points of X . Then $f(x)$ and $f(y)$ are distinct points of Y , because f is injective. Since Y is $*G$ -Hausdorff, there exist two disjoint $*G$ -open sets U and V such that $f(x) \in U$ and $f(y) \in V$. Since f is strongly- $*G$ -continuous and $U \cap V = \emptyset$, we have $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint open sets in X such that $x \in f^{-1}(U)$ and $y \in f^{-1}(V)$. Hence X is Hausdorff space.



Theorem 3.34 : Let X be a space and Y be s -Hausdorff. If $f: X \rightarrow Y$ be $(s, *G)$ -continuous injective, then X is $*G$ -Hausdorff.

Proof : Let x and y be any two distinct points of X . Then $f(x)$ and $f(y)$ are distinct points of Y , because f is injective. Since Y is s -Hausdorff, there exist two disjoint semiopen sets U and V such that $f(x) \in U$ and $f(y) \in V$. Since f is $(s, *G)$ -continuous and $U \cap V = \emptyset$, we have $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint $*G$ -open sets in X such that $x \in f^{-1}(U)$ and $y \in f^{-1}(V)$. Hence X is $*G$ -Hausdorff space.

We define the following.

Definition 3.35: A function $f: X \rightarrow Y$ is said to be $*G$ -open if the image of each open set of X is $*G$ -open in Y .

Definition 3.36: A function $f: X \rightarrow Y$ is said to be always $*G$ -open if the image of each $*G$ -open set of X is $*G$ -open in Y .

Clearly, every always $*G$ -open function is $*G$ -open.

Definition 3.37 : A function $f: X \rightarrow Y$ is said to be $*G$ -closed if the image of each closed set of X is $*G$ -closed in Y .

Definition 3.38: A function $f: X \rightarrow Y$ is said to be always $*G$ -closed if the image of each $*G$ -closed set of X is $*G$ -closed in Y .

Clearly, every always $*G$ -closed function is $*G$ -closed.

We prove the following.

Theorem 3.39: Let $f: X \rightarrow Y$ be a bijection. Then the following are equivalent.

- (i) f is always $*G$ -open.
- (ii) f is always $*G$ -closed.
- (iii) f^{-1} is always $*G$ -continuous function.

Proof: Obvious

Theorem 3.40 : Let $f: X \rightarrow Y$ be always $*G$ -continuous function and $g: Y \rightarrow Z$ be $(g, *G)$ -continuous function, then $g \circ f$ is $(g, *G)$ -continuous function.

Theorem 3.41 : Let $f: X \rightarrow Y$ be always $*G$ -continuous function and $g: Y \rightarrow Z$ be $(s, *G)$ -continuous function, then $g \circ f$ is $(s, *G)$ -continuous function.

Theorem 3.42 : Let $f: X \rightarrow Y$ be always $*G$ -continuous function and $g: Y \rightarrow Z$ be $(G^*, *G)$ -continuous function, then $g \circ f$ is $(G^*, *G)$ -continuous function.

Theorem 3.43 : Let $f: X \rightarrow Y$ be $(g, *G)$ -continuous function and $g: Y \rightarrow Z$ be g -continuous function, then $g \circ f$ is $*G$ -continuous function.

Theorem 3.45: Let $f: X \rightarrow Y$ be $(s, *G)$ -continuous function and $g: Y \rightarrow Z$ be semicontinuous function, then $g \circ f$ is $*G$ -continuous function.

Theorem 3.46 : Let $f: X \rightarrow Y$ be $(G^*, *G)$ -continuous function and $g: Y \rightarrow Z$ be G^* -continuous function, then $g \circ f$ is $*G$ -continuous function.

We, define the following.

Definition 3.47: A function $f: X \rightarrow Y$ is said to be strongly $*G$ -open if the image of each $*G$ -open set of X is open in Y .

Theorem 3.48: Let $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ be any two functions. Then,

- (i) If f is $*G$ -open and g is always $*G$ -open, then $g \circ f$ is $*G$ -open.
- (ii) If f is strongly $*G$ -open and g is $*G$ -open, then $g \circ f$ is always $*G$ -open.
- (iii) If f is always $*G$ -open and g is strongly $*G$ -open, then $g \circ f$ is strongly $*G$ -open.

Proof : (i) Let U be any open set in X , $f(U)$ is $*G$ -open set in Y , because f is $*G$ -open function. Again, g is always $*G$ -open and $f(U)$ is $*G$ -open set in Y , then $g(f(U)) = g \circ f(U)$ is $*G$ -open set in Z . This shows that $g \circ f$ is $*G$ -open function.

(ii) Let U be any $*G$ -open set in X , $f(U)$ is open set in Y , because f is strongly $*G$ -open function. Again, g is $*G$ -open and $f(U)$ is open set in Y , then $g(f(U)) = g \circ f(U)$ is $*G$ -open set in Z . This shows that $g \circ f$ is always $*G$ -open function.

(iii) Let U be any $*G$ -open set in X , $f(U)$ is $*G$ -open set in Y , because f is always $*G$ -open function. Again, g is strongly $*G$ -open and $f(U)$ is $*G$ -open set in Y , then $g(f(U)) = g \circ f(U)$ is open set in Z . This shows that $g \circ f$ is strongly $*G$ -open function.

We, define the following.

Definition 3.49: A function $f: X \rightarrow Y$ is said to be strongly $*G$ -closed if the image of each $*G$ -closed set of X is closed in Y .

Theorem 3.50: Let $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ be any two functions. Then,

- (i) If f is $*G$ -closed and g is always $*G$ -closed, then $g \circ f$ is $*G$ -closed.
- (ii) If f is strongly $*G$ -closed and g is $*G$ -closed, then $g \circ f$ is always $*G$ -closed.
- (iii) If f is always $*G$ -closed and g is strongly $*G$ -closed, then $g \circ f$ is strongly $*G$ -closed.

The routine proof of the Theorem is omitted.

Definition 3.51: A bijection $f: X \rightarrow Y$ is called $*G$ -homeomorphism if f is both $*G$ -continuous function and $*G$ -open function.

Theorem 3.52: Every homeomorphism is an $*G$ -homeomorphism..

We, define the following.

Definition 3.53: A bijection $f: X \rightarrow Y$ is said to be always $*G$ -homeomorphism if both f and f^{-1} are always $*G$ -continuous functions.

Theorem 3.54: Every always $*G$ -homeomorphism is an $*G$ -homeomorphism.

Proof: Obvious.

Finally, we introduce the following new classes of generalized closed sets :

Definition 3.55: A subset A of a space X is called $**G$ -closed set if $g_s Cl(A) \subset U$ whenever $A \subset U$ and U is open set in X .

The complement of a $**G$ -closed set of a space X is called $**G$ -open set in X .

Definition 3.56: A subset A of a space X is called S^* -closed set if $sg Cl(A) \subset U$ whenever $A \subset U$ and U is g -open in X .

The complement of a S^* -closed set of a space X is called S^* -open set in X .

Definition 3.57: A subset A of a space X is called S^{**} -closed set if $sg Cl(A) \subset U$ whenever $A \subset U$ and U is open in X .

The complement of a S^{**} -closed set of a space X is called S^{**} -open set in X .

Definition 3.58: A subset A of a space X is called G^{**} -closed set if $gsp Cl(A) \subset U$ whenever $A \subset U$ and U is open in X .

The complement of a G^{**} -closed set of a space X is called G^{**} -open set in X .

IV. CONCLUSION

At the end of this paper, we introduce the following new classes of generalized closed sets :

$**G$ -closed sets, S^* -closed sets, S^{**} -closed sets and G^{**} -closed sets in topological spaces. Our aim is to study their basic properties in the future papers.

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