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An Optimization Model for the Integration of Location-Allocation-Routing in a Restricted Distribution Network

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ABSTRACT: In the distribution network, it is crucial to identify the location of the facility as suppliers that not only influence the performance of the organization, but also the ability to quickly satisfy clients. The paper discusses the integrated Location-Allocation-Routing issue in such a way as to optimize the overall costs by selecting a subset of candidate installations and designing a number of shipping lines that meet certain constraints at the same time. In this article, we are imposing restrictions on distance and restricted routes. We use an integer programming model to define the problem. A feasible neighbourhood search is introduced in order to solve the outcome model.

KEYWORDS: Routing problem, Integer programming, Feasible search, Distribution network

I. INTRODUCTION

The structure of a distribution network usually requires selecting the best location for facilities and assigning clients to selected installations. With the location-allocation models, this issue can be solved. The aim of these models is to determine from a list of applicants the appropriate areas of facilities so that all travel costs from installations to customers are minimised, and an optimum number of customers in an area of interest must be allocated to meet customer demands. The location of facilities in a distribution network thereby indicates a crucial decision which affects not just the cost-effectiveness of the enterprise but the customer satisfaction capabilities. The word allocation requires guidelines defining how the candidate's demands are allocated. In the location-allocation models there exist three key elements such as client (or demand) position, candidate destination list and distance or duration of the journey across facilities and customer locations.

The challenges with Facility location attracted a lot of scientists and contributed to many issues in the real world. Originally, in [1] indicates a concern with the facility location problem. He proposed a Weber facility location problem, to establish where a warehouse is located so that the length between the warehouse and its customers is minimized. In [2] used the facility location system model in Indian rural public schools to improve regional accessing.

The development of the distribution system includes picking and finding the best destinations for the constructions and assigning the customers to the selected constructions. It is appropriate to overcome this problem with the location-allocation models. The aim of these models is to identify the ideal facilities from the candidate list to minimize the overall cost of transport from facilities to customers and to assign an optimum number of consumers to areas of concern to meet the needs of the customer. Conversely, the establishment of locations in a distribution network is a critical aspect that directly affects not only the earnings of a company but also the potential for customer service. The allocation term includes regulations to indicate how the requirements of the applicant are allocated. In the location-allocation models, there are three main aspects, namely the location (or request) of the customer, the candidate list and the distance or time span between the site and the customer.

Issues of facility location influenced many academics and contributed to other challenges in the modern world. At the beginning, [1] points to a problem with the location of the facility. In order to minimize the average distance between the warehouse and its consumers, Weber has a problem with the location of the facility to determine where the warehouse is located. In [2] have used the facility location model to improve access to public schools in rural India. Location allocation models hold a vital role in the development of health facilities, including the location of the best service locations in the new area, the assessment of the effectiveness of past localization decisions and the improvement of existing localization patterns [3]. The authors include an interesting description of location-allocation literature using location-allocation models of health planning in developing countries. In [4] also used this pattern in the

City of Lujan, Argentina, to identify critical healthcare centers. In order to enhance the strategic planning of public health providers by Polo et al. [5] combines the location-allocation paradigm with accessibility.

The P-median problem is a simple approach used for location-allocation models. The median points between potential installations are determined by the situation, which reduces total costs to a minimum [6]. The problem is also identified in the literature as one approach to location-allocation problems. The aim of this issue is to locate facilities that provide services to people with access to facilities over a specific radius.

Location modeling transactions are carried out on out-of-back lanes that are required to be visited by one individual customer or, more commonly, by the customer who reaches the facility individually. As a result, shipping costs are independent of other deliveries. In some cases, however, several stops are made available to two or more customers; in this case, the cost of the service depends on the route and order of the visits made to certain customers. For the sake of calculate reliably the cost of multiple stops in the localization model, the routing problem must be addressed at the same time as the position problems. This type of problem is referred to as the problem of location-routing.

The aim is often to choose a location from a subset of the candidate facilities and to construct a number of routes to meet the requirements of the location-routing problem (LRP):

1. Customer requests without exceeding the capacity of the vehicle or facility.
2. The number of vehicles, the length of the route and the duration of the route;
3. Each path begins and ends at the same location.

The location-routing problems are specifically related to both the traditional location issue and the problem of vehicle routing. In fact, both of these latter problems can be considered as special cases of LRP. The LRP is a typical location issue if we want to connect all customers directly to a single warehouse. When the repository sites, on the other hand, are fixed, the LRP decreases to the VRP. In a realistic context, location-routing is part of distribution management and can typically be viewed as a combination optimization issue from a mathematical point of view. We recognize that this is an NP-hard problem because it involves two NP-hard issues (facility location and vehicle routing). Since there are several versions of the problems, not all of the formulations can be reproduced here. The reader is referred to [7] for a very good review of the various formulations in the first instance.

Most of the work to date focuses on heuristic methods as two NP-hard problems are combined between LRPs. Generally speaking, heuristics breaks down the problem into its three components, the location of the facility, the allocation of customers to the equipment and the routing of vehicles and resolves a number of famous concerns such as p-median, location-allocation and vehicle routing. Accurate processes have been developed for a few LRP models derived from two-index vehicle routing problem (VRP). By constraint relaxation method, in [8] made a single depot model.

Laporte [9] proposes a framework equivalent to the model where the number of vehicles used in the model varies. In [10] solve a multi-depot problem that usually includes p facilities. Seven candidate facilities and 40 consumers are the main issues. In [10] uses a constraint relaxation approach to overcome a multi-depot LRP capability. Eight candidate installations and 20 customers are the biggest problem they have solved in order to make their jobs optimal. For the solution of asymmetric LRPs, which includes three candidate installations and 80 customers by Laporte et al. [11] apply a branch and bound procedure. Two Meta-Heuristic Genetic and Tab Analysis Algorithms are used by Gharavani and Setak [12]. Since its parameters have a significant impact on the performance of these heuristic algorithms, the Taguchi Method is used to set the parameters of the developed algorithms. In [13] use multi-objective optimization for location-allocation of web service problem. Recently, in [14] propose a linear integer programming model for solving location-allocation problem in transportation of hazardous materials.

It is also prohibited to travel with pairs of edges which may take place dynamically due to restrictions on peak hours, road blocks, construction, etc. There are less common prohibited sub-paths, but they may arise, e.g. if heavy traffic prevents it from turning left shortly after entering a multi-lane road from the right. It is more reasonable to find a detour from the point of failure if we drive a single vehicle when a prohibited route is found.

The combination location-allocation-routing problem model is discussed in this study. We enforce certain restrictions, i.e. distance and the prohibited path. The combined problem can be defined as a large-scale numerical system. The concept is overcome by using the exact method called the Feasible Neighborhood Search Approach.

II. LOCATION-ALLOCATION MODEL

The P-median problem lies in the basic forms of allocation models for the private sector. The model is structured to reduce the overall distance between consumers and the nearest service centers.

The following notations are defined.

Set

I Set of customer nodes

J Set of potential facility sites

M Number of customer points in the considered area

N Number of potential facility locations

Parameters

a_i Demand at node $i \in I$

d_{ij} Distance between node $i \in I$ and $j \in J$

Q Number of facilities to be located

Variables

X_{ij} Binary variable whether customer $i \in I$ is assigned to a facility $j \in J$

The model can be formulated as follows.

The objective is to minimize the total distance or travel time between customer node i and facility site node j.

$$Minimize \sum_{i \in I} \sum_{j \in J} a_{ij} d_{ij} x_{ij} \tag{1}$$

There are constraints need to be satisfied.

In order to make sure that every customer (or demand) is assigned to one and only one facility, we need the following expression.

$$\sum_{j \in J} x_{ij} = 1, \qquad \forall i \in I \tag{2}$$

$$x_{ij} \leq x_{jj}, \qquad \forall i \in I, \forall j \in J \tag{3}$$

The next equation is to limit the number of facilities to be located

$$\sum_{j \in J} x_{jj} = Q \tag{4}$$

III. LOCATION-ROUTING MODEL

Next, we are showing an LRP method based on distance partitioning. The aim is to select a number of locations and to build a number of associated delivery routes in order to minimize installation costs plus routing costs. The package of routes must ensure that every customer is visited exactly on a single route and that the length of each route does not exceed the maximum distance.

Let I be the set of customer location nodes and J be the set of candidate facility location nodes. We define the graph $G=(N,A)$, where $N = I \cup J$ is the set of nodes and $A=N\times N$ is the set of arcs. We let d_{ij} for all $(i, j)\in A$ be the distance between nodes i and j. The distances satisfy the triangle inequality. The distances satisfy the triangle inequality. For applications in which the distance constraint applies to the length of the route to the last customer instead of the length of the return trip to the depot, we set d_{ij} to 0 for all (i, j) with $i\in I$ and $j \in J$. We define a feasible route k associated with facility j as a simple circuit that begins at facility j, visits one or more customer nodes and returns to facility j and that has a total distance of at most the maximum distance, denoted M. Then, we let P_j denote the set of all feasible routes associated with the facility j for all $j \in J$. The cost of a route $k \in P_j$ is the sum of the costs of the arcs in the route. The cost of an arc $(i, j) \in A$ is proportional to the distance d_{ij} to reflect distance related operating costs.

Parameters

$$a_{ijk} = \begin{cases} 1, & \text{if route k associated with facility } j \text{ visits customer } i, \forall i \in I, \forall j \in J, \forall k \in P_j \\ 0, & \text{otherwise} \end{cases}$$

c_{jk} cost of route k associated with facility $j, \forall j \in J, \forall k \in P_j$

f_j fixed cost associated with selecting facility $j, \forall j \in J$

$\propto$ object weighted factor

Decision Variables

$$X_j = \begin{cases} 1, & \text{if facility } j \text{ is selected, } \forall j \in J \\ 0, & \text{otherwise} \end{cases}$$

$$Y_{jk} = \begin{cases} 1, & \text{if route } k \text{ associated with facility } j \text{ is selected, } \forall j \in J, \forall k \in P_j \\ 0, & \text{otherwise} \end{cases}$$

The objective is to minimize cost

$$\text{Minimize} \quad \alpha.\sum_{j\in J} f_j X_j + \sum_{j\in J}\sum_{k\in P_j} c_{jk} Y_{jk} \tag{5}$$

$$\text{s.t.} \quad \sum_{j\in J}\sum_{k\in P_j} a_{ijk} Y_{jk} = 1 \quad \forall i \in I \tag{6}$$

$$X_j - Y_{jk} \ge 0 \quad \forall j \in J, \forall k \in P_j \tag{7}$$

$$X_j \in \{0,1\} \quad \forall j \in J \tag{8}$$

$$Y_{jk} \in \{0,1\} \quad \forall j \in J, \forall k \in P_j \tag{9}$$

The objective function (1) seeks to minimize the weighted sum of the facility costs and the routing costs. Constraints (2) are the set partitioning constraints that require each customer i be served by exactly one of the selected routes. Constraints (3) require that facility j be selected if a route k associated with facility j is selected. Constraints (4) and (5) are standard binary restrictions. The LRP with distance constraints is NP-hard. By placing very large costs on the arcs connecting two customer nodes, we obtain a special case of the model in which the selected routes contain exactly one customer.

As presented, the formulation LRP potentially contains an exponential number of variables y_{jk} and an exponential number of constraints (3). Thus, for instances of practical size, enumerating all of the feasible routes and solving the resulting integer program is unlikely to be effective. Instead, we will use feasible neighbourhood search for solving the model.

IV. FORBIDDEN ROUTE

We are given an directed graph $G(V, A)$ with $n = |V|$ vertices and $m = |A|$ edges where each edge $e \in A$ has a positive weight denoting its length. We are also given a source vertex $s \in V$, a destination vertex $t \in V$, and a set X of forbidden route in G. The graph G together with X models a vehicle routing network in which a vehicle cannot follow any route in X because of the physical constraints . We want to find a shortest route from s to t that does not contain any route in X as a subpath—we make the goal more precise as follows. A route is a sequence of vertices each joined by an edge to the next vertex in the sequence. Note that we allow a route to visit vertices and edges more than once. If a route does not visit any vertex more than once, we explicitly call it a simple route. A simple directed route from vertex v to vertex w in G is called a forbidden route or an exception if a vehicle cannot follow the route from v to w because of the physical constraints. Given a set X of forbidden route, a route $(v_1, v_2, v_3, \ldots, v_l)$ is said to avoid A if $(v_i, v_i + 1, \ldots, v_j) \notin A$ for all i, j such that $1 \leq i < j \leq l$.

V. LOCATION-ALLOCATION-ROUTING PROBLEM WITH DISTANCE AND FORBIDDEN ROUTE

5.1 Problem formulation

Given a set of products L need to be distributed to a set of suppliers. The company has determined a list of candidate as potential suppliers (J). There is a set of customer nodes I with given demands spread across the city. A set of vehicle (M) is available to deliver the product. Each vehicle has a maximum capacity, Q. As mentioned in the problem description of location-routing model, we define a feasible route r associated with facility j as a simple directed graph that begins at facility j, visits one or more customer nodes and returns to facility j. with maximum distance of travelling N. Then, we let P_j denote the set of all feasible routes associated with the facility j for all $j \in J$. Unfortunately, due to physical constraint, there are forbidden route in which a vehicle cannot pass by.

5.2 The Model

The Location-Allocation-Routing Problem can be formulated mathematically as follows.
Notations used.

Sets

L Set of product
J Set of potential suppliers
I Set of customers' nbode
M Set of vehicles
R Set of feasible route
X Set of forbidden route

Parameters

a_i Demand at node $i \in I$

d_{ij} Distance from node $i \in I$ to node $j \in J$

Q Maximum weight capacity of a vehicle

q_{ijrm} Weight demand of customer i delivered from location j of vehicle m using route r

λ, ρ Costs

γ^l_{ijrm} Cost of transportation of vehicle m to deliver product l from supplier j to customer i using route r

Variables

x_{ij} Binary variable whether supplier j will serve customer i

y^l_j Binary variable if product l is located to supplier j

z^l_{ijrm} Binary variable if product l will be delivered to customer i from supplier j through route r using vehicle m

The objective function of this model is to minimize total cost.

$$\lambda \sum_{j \in J} \sum_{l \in L} y_j^l + \rho \sum_{i \in I} \sum_{j \in J} d_{ij} a_i x_{ij} + \sum_{i \in I} \sum_{j \in J} \sum_{r \in R, r \notin X} \sum_{l \in L} \gamma_{ijrm}^l z_{ijrm}^l \tag{10}$$

Subject to constraints

The following expression is to make sure that every customer is assigned to one and only one supplier.

$$\sum_{j \in J} x_{ij} = 1, \qquad \forall i \in I \tag{11}$$

$$x_{ij} \le x_{jj}, \qquad \forall i \in I, \forall j \in J \tag{12}$$

The next constraint is to guarantee that product $l \in L$ is only located at supplier $j \in J$

$$\sum_{j \in J} y_j^l = 1, \qquad \forall l \in L \tag{13}$$

Eq. (14) presents the requirement that each customer i is served exactly by one of the selected routes but not the forbidden routes.

$$\sum_{i \in I} \sum_{j \in J} \sum_{r \in R, r \notin X} b_{ijrm} z_{ijrm}^l = 1, \qquad \forall l \in L, \forall m \in M \tag{14}$$

Constraints (15) state that supplier j be selected if a route r, as long as $r \notin X$, associated with supplier j is selected.

$$y_j^l - z_{ijrm}^l \ge 0, \qquad \forall i \in I, \forall j \in J, \forall r \in R, \forall r \notin X, \forall l \in L, \forall m \in M \tag{15}$$

Constraints (16) guarantee that vehicle capacities are respected in weight.

$$\sum_{i \in I} \sum_{j \in J} \sum_{r \in R, r \notin X} q_{ijrm} z_{ijrm}^l \le Q_m \qquad \forall l \in L, \forall m \in M \tag{16}$$

$$x_{ij}, y_j^l, z_{ijrm}^l \in \{0,1\} \qquad \forall i \in I, \forall j \in J, \forall m \in M, \forall r \in R, \forall l \in L \tag{17}$$

The model is a large scale Integer programming problem.

We develop the following method for solving the model

VI. NEIGHBOUHOOD SEARCH

It should be noted that the reduced gradient vector, which is usually used to detect an optimal condition, is generally not available in integer programming, even if problems are convex. We will therefore enforce a certain requirement for the local testing search process to ensure that the "best" possible integer solution has been achieved.

In [15] proposed a quantity test in order to replace the price test for optimality in integer programming. The evaluation shall be carried out by checking the feasible neighborhood element, under which a neighboring point is also feasible and which improves the objective function.

Let $[\beta]_k$ be an integer point belongs to a finite set of neighbourhood $N([\beta]_{k)}$. We define a neighbourhood system associated with $[\beta]_k$, that is, if such an integer point satisfies the following two requirements

1. If$\left[\beta\right]_j \in N([\beta]_k)$ then $[\beta]_k \in [\beta]_j), j \neq k.$
2. $N([\beta]_k) = [\beta]_k + N(0)$

With respect to the neighbourhood system mentioned above, the proposed integerizing strategy can be described as follows.

Given a non-integer component, x_k, of an optimal vector , x_B. The adjacent points of x_k, being considered are $[x_k]$ and $[x_k] + 1$. If one of these points satisfies the constraints and yields a minimum deterioration of the optimal objective value we move to another component, if not we have integer-feasible solution.

Let $[x_k]$ be the integer feasible point which satisfies the above conditions. We could then say if $[x_k] + 1 \in N([x_k])$ implies that the point $[x_k] + 1$ is either infeasible or yields an inferior value to the objective function obtained with respect to $[x_k]$. In this case $[x_k]$ is said to be an "optimal" integer feasible solution to the integer programming problem. Obviously, in our case, a neigbourhood search is conducted through proposed feasible points such that the integer feasible solution would be at the least distance from the optimal continuous solution.

VII. THE IDEA OF THE METHOD

Before we proceed to the case of MINLP problems, it is worthwhile to discuss the basic strategy of process for linear case, i.e., Mixed Integer Linear Programming (MILP) problems.

Consider a MILP problem with the following form

$$\text{Minimize } P = c^T x \tag{18}$$

$$\text{Subject to } Ax \leq b \tag{19}$$

$$x \geq 0 \tag{20}$$

$$x_j \text{ integer for some } j \in J \tag{21}$$

A component of the optimal basic feasible vector $(x_B)_k$, to MILP solved as continuous can be written as

$$(x_B)_k = \beta_k - \alpha_{k1}(x_N)_1 - \cdots - \alpha_{kj}(x_N)_j - \cdots \alpha_{kn} - m(x_N)_N n \tag{22}$$

Note that, this expression can be found in the final tableau of Simplex procedure. If $(x_B)_k$ is an integer variable and we assume that β_k is not an integer, the partitioning of β_k into the integer and fractional components is that given

$$[\beta_k] + f_k\,, \;\; 0 \leq f_k \leq 1 \tag{23}$$

suppose we wish to increase $(x_B)_k$ to its nearest integer, $([\beta] + 1)$. Based on the idea of suboptimal solutions we may elevate a particular nonbasic variable, say $(x_N)_{j^*}$, above its bound of zero, provided α_{kj^*}, as one of the element of the vector α_{j^*}, is negative. Let Δ_{j^*} be amount of movement of the non variable $(x_N)_{j^*}$, such that the numerical value of scalar $(x_B)_k$ is integer. Referring to Eqn. (22), Δ_{j^*} can then be expressed as

$$\Delta_{f^*} = \frac{1 - f_k}{-\alpha_{kj^*}} \tag{24}$$

while the remaining nonbasic stay at zero. It can be seen that after substituting (23) into (24) for $(x_N)_{j^*}$ and taking into account the partitioning of β_k given in (23), we obtain

$$(x_B)_k = [\beta] + 1$$

Thus, $(x_B)_k$ is now an integer.

It is now clear that a nonbasic variable plays an important role to integerized the corresponding basic variable. Therefore, the following result is necessary in order to confirm that must be a non-integer variable to work with in integerizing process.

Theorem 1. *Suppose the MILP problem* (18)-(21) *has an optimal solution, then some of the nonbasic variables.* $(x_N)_j, j = 1, \ldots, n$, *must be non-integer variables.*

Proof:
Solving problem as a continuous of slack variables (which are non-integer, except in the case of equality constraint). If we assume that the vector of basic variables consists of all the slack variables then all integer variables would be in the nonbasic vector x_N and therefore integer valued.

7.1 Exploration of the method

It is clear that the other components, $(x_B)_{i\neq k}$, of vector x_B will also be affected as the numerical value of the scalar $(x_N)_{j^*}$ increases to Δ_{j^*}. Consequently, if some element of vector α_{j^*}, i.e., α_{j^*} for $i\neq k$, are positive, then the corresponding element of x_B will decrease, and eventually may pass through zero. However, any component of vector x must not go below zero due to the non-negativity restriction. Therefore, a formula, called the minimum ratio test is needed in order to see what is the maximum movement of the nonbasic $(x_N)_{j^*}$ such that all components of x remain feasible. This ratio test would include two cases.

1. A basic variable $(x_B)_{i\neq k}$ decreases to zero (lower bound) first.
2. The basic variable, $(x_B)_k$ increases to an integer.

Specifically, corresponding to each of these two cases above, one would compute

$$\theta_1 = \min_{i\neq k|\alpha_{j^*}>0}\left\{\frac{\beta_i}{\alpha_{j^*}}\right\} \tag{25}$$

$$\theta_2 = \Delta_{j^*} \tag{26}$$

How far one can release the nonbasic $(x_N)_{j^*}$ from its bound of zero, such that vector x remains feasible, will depend on the ratio test θ^* given below

$$\theta^* = \min(\theta_1, \theta_2) \tag{27}$$

obviously, if $\theta^* = \theta_1$, one of the basic variable $(x_B)_{i\neq k}$ will hit the lower bound before $(x_B)_k$ becomes integer. If $\theta^* = \theta_2$, the numerical value of the basic variable $(x_B)_k$ will be integer and feasibility is still maintained. Analogously, we would be able to reduce the numerical value of the basic variable $(x_B)_k$ to its closest integer $[\beta_k]$. In this case how far the movement of a particular nonbasic variable, $(x_N)_{j^*}$, corresponding to any positive element of vector $\alpha_{j'}$, is given by

$$\Delta_{f'} = \frac{f_k}{\alpha_{kj}} \tag{28}$$

In Linear Programming (LP) terminology the operation conducted in Eqns. (22) and (23) is called the pricing operation. The vector of reduced costs d_j is used to measure the deterioration of the objective function value caused by releasing a nonbasic variable from its bound. Consequently, in deciding which nonbasic should be released in the integerizing process, the vector d_j must be taken into account, such that deterioration is minimized. Recall that the minimum continuous solution provides a lower bound to any integer-feasible solution. Nevertheless, the amount of movement of particular nonbasic variable as given in Eqns. (11) or (15), depends in some way on the corresponding element of vector α_j. Therefore it can be observed that the deterioration of the objective function value due to releasing a nonbasic variable $(x_N)_{j^*}$ so as to integerize a basic variable $(x_B)_k$ may be measured by the ration

$$\left|\frac{d_k}{\alpha_{kj^*}}\right| \tag{29}$$

where $|a|$ means the absolute value of scalar a.

In order to minimize the detonation of the optimal continuous solution we then use the following strategy for deciding which nonbasic variable may be increased from its bound of zero, that is,

$$\min_j\left\{\left|\frac{d_k}{\alpha_{kj^*}}\right|\right\}, \qquad j = 1,\dots,n-m \tag{30}$$

From the "active constraint" strategy and the partitioning of the constraints corresponding to basic(B), superbasic (S) and nonbasic (N) variables we can write

$$\begin{bmatrix} B & S & N \\ 0 & 0 & I \end{bmatrix}\begin{bmatrix} x_b \\ x_N \\ x_S \end{bmatrix} = \begin{bmatrix} b \\ b_N \end{bmatrix} \tag{31}$$

or

$$Bx_b + Sx_N + Nx_S = b \tag{32}$$

$$x_N = b_N \tag{33}$$

The basis matrix B is assumed to be square and nonsingular, we get

$$x_B = \beta - Wx_S - \alpha x_N \tag{34}$$

where

$$\beta = B^{-1}b \tag{35}$$

$$W = B^{-1}S \tag{36}$$

$$\alpha = B^{-1}N \tag{37}$$

Expression (33) indicates that the nonbasic variables are being held equal to their bound. It is evident through the "nearly" basic expression of Eqn. (34), the integerizing strategy discussed in the previous section, designed for MILP problem can be implemented. Particularly, we would be able to release a nonbasic variable from its bound, Eqn.(33) and exchange it with a corresponding basic variable in the integerizing process, although the solution would be degenerate. Furthermore, the Theorem (1) above can also be extended for MINLP problem.

Theorem 2. *Suppose the MINLP problem has a bounded optimal continuous solution, then we can always get a non-integer y_j in the optimum basic variable vector.*

Proof.
1. If these variables are nonbasic, they will be at their bound. Therefore they have integer value.
2. If a y_j is superbasic, it is possible to make y_j basic and bring in a nonbasic at its bound to replace it in the superbasic.

Currently, we are in a position where particular basic variable, $(x_B)_k$ is being integerized, thereby a corresponding nonbasic variable, $(c_N)_{j^*}$, is being released from its bound of zero. Suppose the maximum movement of $(x_N)_{j^*}$ satisfies

$$\theta^* = \Delta_{j^*}$$

such that $(x_B)_k$ is integer valued to exploit the manner of changing the basis in linear programming, we would be able to move $(x_N)_{j^*}$ into B (to replace $(x_B)_k$) and integer-valued $(x_B)_k$ into S in order to maintain the integer solution. We now have a degenerate solution since a basic variable is at its bound. The integerizing process continues with a new set $[B, S]$. In this case, eventually we may end up with all of the integer variables being superbasic.

Theorem 3. *A suboptimal solution exists to the MILP and MINLP problem in which all of the integer variables are superbasic.*

Proof.
1. If all of the integer variables are in *N*, then they will be a bound.
2. If an integer variable is basic it is possible to either
   - Interchange it with a superbasic continuous variable, or
   - Make this integer variable superbasic and bring in a nonbasic at its bound to replace it in the basis which gives a degenerate solution.

The other case which can happen is that a different basic variables $(x_B)_{i\neq k}$ may hit its bound before $(x_B)_k$ becomes integer. Or in other words, we are in a situation where

$$\theta^* = \Delta_1$$

In this case we move the basic variable $(x_B)_j$ into N and its position in the basic variable vector would be replaced by nonbasic $(x_B)_{j^*}$. Note $(x_B)_k$ is still a non-integer basic variable with a new value.

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VIII. CONCLUSION

This article sets out a problem model Location-Allocation-Routing in which distance is taken into account and where any prohibited route occurs. The model framework is based on the problem of location-location, location-routing and VRP with time windows with a prohibited route. Instead, we are eliminating the prohibited road from the previous allocated road. We have solved the problem by using a feasible search for the neighbourhood.

REFERENCES

[1] A. Weber, "Uber den Standort der Industrien", Tubingen Theory of Location of Industries, University of Chicago Press, 1909

[2] V. Tewari, and S. Jena. "High school location decision making in rural India and location-allocation models", In Gosh, A. & Rushton, G. Spatial Analysis and location-allocation models. New York: Van Nostrand Reinhold Company Inc 137-162, 1987

[3] M. Jamshidi, "Median location problem", In Facility Location, R. Z. Farahani and M. Hekmatfar, Eds. Ed: Physica-Verlag HD, pp. 177-191, 2009

[4] G. Buzai, "Location-allocation models applied to urbal public services", Spatial analysis of primary health care centers in the city of Lujan, Argentina. Hungerian Geographical Bulletin 62(4): 387-408, 2013

[5] G. Polo, C. M. Acosta, F. Ferreira, and R. A. Dias, "Location-allocation and accessibility models for improving the spatial planning of public health services", PLOS ONE, March 16, 2015

[6] S. Rahman, and D. Smith, "Use of location-allocation models in health service development planning in developing nations", European Journal of Operations Research 123: 437-452, 2000.

[7] M. Albareda-Sambola, A. J. Diaz, and E. Fernandez, "A compact model and tight bounds for a combined location routing problem", Computers & Operations Research, V 32, n 4, pp. 407-428, 2005

[8] G Laporte and Y Nobert, " An exact algorithm for minimizing routing and operating cost in depot location", European Journal of Operational Research, **6** 224–226, 1981

[9] G Laporte, "Location routing problems in golden B, Assad A (eds) Vehicle Routing: Methods and Studies (North-Holland: Amsterdam) pp 293–318, 1988

[10] G Laporte, Y Nobert, and D Arpin, " An exact algorithm for solving a capacitated location-routing problem", Annals of Operations Research **6** 293–310, 1986

[11] G Laporte, Y Nobert, and Y Pelletier, "Hamiltonian location problems", European Journal of Operational Research **12** 82–89, 1983

[12] M. Gharavani and M. Setak, "A capacitated location-routing problem with semi soft time windows", Advanced Computational Techniques in Electromagnetic, No. 1(2015), 26-40, 2015

[13] B. Tan, H. Ma, Y. Mei, M. Zhang, "Evolutionary multi-objective optimization for web-service location allocation problem", IEEE Transactions on Services Computing, to be published. doi: 10.1109/TSC.2018.2793266.

[14] Fan, J., Yu, L., Li, X., Shang, C., & Ha, M, "Reliable location allocation for hazardous materials", Information Sciences, (2019), 501, 688-707. https://doi.org/10.1016/j.ins.2019.03.006

[15] H E Scarf, "Testing for optimality in the absence of convexity in Walter P Heller", Ross M Starr and David A Starrett (Eds) (Cambridge University Press) pp 117-134, 1986