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Reliable Retrial $M^X/G/1$ Queueing System under Multiple Adapted Vacation Policy

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ABSTRACT: In the present paper, a batch arrival single server retrial queueing system in which the server takes vacations whenever the system becomes empty following the Multiple Adapted Vacation (MAV) policy is considered. The performance measures such as queue size probabilities when the system is in different states and their corresponding mean queue lengths are calculated. The stochastic decomposition property of vacation models is also verified. The total probability generating function for different vacation models are deduced from this single model.

KEYWORDS: Retrial, Multiple Adapted Vacation, Stochastic Decomposition property.

I. INTRODUCTION

Server vacation in queueing models represents the period of temporary server's absence. Extensive studies have been done by several authors on the vacation models in the past five decades. Excellent surveys on the earlier works of vacation have been done by Doshi (1986). It was then surveyed by Tagaki (1991), Tian and Zhang (2006) and Ke, Wu and Zhang (2010). A wide class of vacation policies for governing vacation mechanism has been discussed in the literature and various authors have analysed the vacation queueing models by considering different types of vacation as independent characteristics.

Retrial queueing systems are characterised by the fact that a primary request, finding all servers and waiting positions busy upon arrival, leaves the service area and repeats its demand after some random time. Thus the retrial queues can be considered as an alternative to queues with losses that do not take repeated attempts into account. For a review of main results and methods, the reader is referred to the specific monographs by Falin and Templeton (1997). Aissani (1998) discussed the retrial $M^X/G/1$ queueing models with exhaustive vacations. Mytalas and Zazanis (2015) considered a more general MAV policy controlled by a sequence of probabilities $\{\beta_j\}$ for classical queues. MAV policy combines all the idle vacation types into a single model. The purpose of the paper is to consider a retrial queueing system with batch arrivals in which the idle time of the server is utilized for vacations following MAV policy so that the corresponding results for the queueing model of various vacation types including the non-vacation type can be deduced from the single model.

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II. MATHEMATICAL ANALYSIS OF THE SYSTEM

2.1 Model Description

Arrival Pattern: Customers arrive in batches at the system according to a time homogeneous Poisson process with group arrival rate λ . The batch size \mathbf{X} is a random variable with probability distribution $\Pr(\mathbf{X} = k) = g_k$, $k = 1, 2, \dots$ and $\sum_{k=1}^{\infty} g_k = 1$. There is no waiting space in front of the server. If an arriving batch finds the server is idle, then one of the arriving customers begins to receive service and the other customers leave the service area and enter a group of blocked customers called “Orbit” according to **FCFS** discipline. On completion of the service, the customer at the head of the retrial queue competes with potential primary customers to decide which customer will enter the next service. If a batch of primary customers arrives first, the retrial customer may cancel its attempt for service and returns to its position. The retrial time A is assumed to follow arbitrary distribution function $A(x)$ and density function $a(x)$ with finite moments.

Multiple Adapted Vacation (MAV) Policy: A cycle starts whenever the system becomes empty and the server is deactivated. The deactivated server either remains idle in the system with probability $1 - \beta_0$ or takes a (first) vacation with probability β_0 . Upon returning from each vacation j ($j = 1, 2, \dots$), if the server finds at least one customer waiting in the orbit, then he immediately joins the system and waits for the retrial of the customer. Otherwise if there is no customer found waiting in the queue, then the server either joins the system with probability $1 - \beta_j$ or takes a new vacation with probability β_j . This process continues until a customer arrives and then the server starts a new busy cycle. The time during which the server is either on vacation or idle in the system is called idle period.

Busy Period: Busy period starts at the end of each idle period. During busy period, the server provides service to the customers one at a time according to **FCFS** queue discipline. Busy period ends when the system becomes empty again.

This model is denoted by the symbol $\mathbf{M}^{\mathbf{X}}/\mathbf{G}/1/\mathbf{MAV}$. Various stochastic processes involved in the queueing system are assumed to be independent of each other. The vacation time VI and service time S are assumed to follow general distributions $VI(x)$ and $S(x)$; density functions $vI(x)$ and $s(x)$ respectively with finite moments.

If $f(x)$ is the density function of the probability distribution $F(x)$, then the LST is $F^*(\theta) = \int_0^{\infty} e^{-\theta x} d(F(x))$. The LST of the random variables A , VI and S are denoted by $A^*(\theta)$, $VI^*(\theta)$ and $S^*(\theta)$ respectively. The system is analysed using Supplementary variable technique, by introducing the remaining times of the random variables as supplementary variables.

Let $A^0(t)$, $VI^0(t)$ and $S^0(t)$ denote the remaining times of the random variables namely retrial time, idle vacation time and service time at time t respectively. Further different states of the server at time t are denoted by $Y(t) = \{0, 1, 2\}$ which respectively denotes idle state, vacation state during idle period and busy state. The supplementary variables are introduced in order to obtain a bivariate Markov process $\{N(t), \delta(t)\}$ where $N(t)$ denotes the queue size random variable and $\delta(t) = (A^0(t), VI^0(t), S^0(t))$ according as $Y(t) = (0, 1, 2)$ respectively.

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Let $PI_n(w,t)dt = \Pr\{N(t) = n, w \leq A^0(t) \leq w + dt, Y(t) = 0, n \geq 1\}$ be the joint probability that at time t , there are n customers in the retrial orbit, the server is idle and the remaining retrial time of the server is between w and $w + dt$, where $n \geq 1$ and $PI_0(t) = \Pr\{N(t) = 0, Y(t) = 0\}$ be the probability that the server is idle at time t , and there is no customer in the retrial orbit.

Let $QI_{n,j}(x,t)dt = \Pr\{N(t) = n, x \leq VI^0(t) \leq x + dt, Y(t) = 1, n \geq 0\}$ be the joint probability that at time t , there are n customers in the retrial orbit, the server is in j^{th} vacation and the remaining vacation time of the server is between x and $x + dt$, where $n \geq 0$.

Let $P_n(x,t)dt = \Pr\{N(t) = n, x \leq S^0(t) \leq x + dt, Y(t) = 2, n \geq 1\}$ be the joint probability that at time t , there are n customers in the retrial orbit, the server is busy, a customer is being served in service and the remaining service time lies between x and $x + dt$, where $n \geq 1$.

Let $E(Y^k)$ ($k=1,2,\dots$) denote the k^{th} moment of the random variable Y .

Assuming that the steady state probabilities exist, we have as $t \rightarrow \infty$, the probabilities are independent of time t . Let $PI_n(w)$, $P_n(x)$ and $QI_{n,j}(x)$ respectively denote that the server is idle in the empty system, stays on vacation and busy at steady-state and $PI_n(\theta)$, $P_n(\theta)$ and $QI_{n,j}(\theta)$ denote LST of the corresponding probabilities. Thus the LST of the Steady State Queue Size Equations are obtained for the model. The following partial generating functions are introduced to analyse the model.

$$PI^*(z,\theta) = \sum_{n=1}^{\infty} PI_n^*(\theta) z^n \quad \quad \quad PI(z,0) = \sum_{n=1}^{\infty} PI_n(0) z^n$$

$$P^*(z,\theta) = \sum_{n=0}^{\infty} P_n^*(\theta) z^n \quad \quad \quad P(z,0) = \sum_{n=0}^{\infty} P_n(0) z^n$$

$$QI_j^*(z,\theta) = \sum_{n=0}^{\infty} QI_{n,j}^*(\theta) z^n \quad \quad \quad QI_j(z,0) = \sum_{n=0}^{\infty} QI_{n,j}(0) z^n$$

2.2 LST of the Steady State Queue Size Equations

$$\lambda PI_0 = \sum_{j=1}^{\infty} (1 - \beta_j) QI_{0,j}(0) + P_0(0) (1 - \beta_0) \quad (1)$$

$$\theta PI_n^*(\theta) - PI_n(0) = \lambda PI_n^*(\theta) - \sum_{j=1}^{\infty} QI_{n,j}(0) A^*(\theta) - P_n(0) A^*(\theta), \quad n \geq 1 \quad (2)$$

$$\theta P_0^*(\theta) - P_0(0) = \lambda P_0^*(\theta) - PI_1(0) S^*(\theta) - \lambda g_1 PI_0 S^*(\theta) \quad (3)$$

$$\theta P_n^*(\theta) - P_n(0) = \lambda P_n^*(\theta) - PI_{n+1}(0) S^*(\theta) - \lambda S^*(\theta) \int_0^{\infty} \sum_{k=1}^n PI_{n-k+1}(w) dw g_k - \lambda \sum_{k=1}^n P_{n-k}^*(\theta) g_k - \lambda g_{n+1} PI_0 S^*(\theta), \quad n \geq 1 \quad (4)$$

$$\theta QI_{0,1}^*(\theta) - QI_{0,1}(0) = \lambda QI_{0,1}^*(\theta) - \beta_0 P_0(0) VI^*(\theta) \quad (5)$$

$$\theta QI_{0,j}^*(\theta) - QI_{0,j}(0) = \lambda QI_{0,j}^*(\theta) - \beta_{j-1} QI_{0,j-1}(0) VI^*(\theta), \quad j \geq 2 \quad (6)$$

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$$\theta QI_{n,j}^*(\theta) - QI_{n,j}(0) = \lambda QI_{n,j}^*(\theta) - \lambda \sum_{k=1}^n QI_{n-k,j}^*(\theta) g_k, \quad n \geq 1, j \geq 1 \quad (7)$$

2.3 Probability Generating Functions

Using algebraic manipulations the partial probability generating functions of the queue size probabilities at arbitrary epoch when the server is in different states are obtained in terms of $P_0(0)$ and are listed below:

$$QI^*(z,0) = P_0(0) \sum_{j=0}^{\infty} \alpha_0^j \left(\prod_{i=0}^j \beta_i \right) \frac{1 - VI^*(w_x(z))}{w_x(z)} \quad (8)$$

$$PI^*(z,0) = P_0(0) Y(z) \frac{1 - A^*(\lambda)}{\lambda} \quad (9)$$

$$P^*(z,0) = P_0(0) Q(z) \frac{1 - S^*(w_x(z))}{w_x(z)} \quad (10)$$

where $w_x(z) = \lambda(1 - X(z))$

$$VI^*(w_x(z)) = \sum_{n=0}^{\infty} \alpha_n z^n \text{ implies } \alpha_n = \int_0^{\infty} e^{-\lambda t} \sum_{i=0}^{\infty} \frac{(\lambda t)^i g_n^{(i)}}{i!} d(vI(t))$$

$$\varphi = \sum_{j=1}^{\infty} (1 - \beta_j) \alpha_0^j \left(\prod_{i=0}^{j-1} \beta_i \right) + (1 - \beta_0) \quad (11)$$

$$I_0(z) = \varphi + \sum_{j=0}^{\infty} \alpha_0^j \left(\prod_{i=0}^j \beta_i \right) (1 - VI^*(w_x(z))) \quad (12)$$

$$M_1(z) = A^*(\lambda) + X(z) (1 - A^*(\lambda)) \quad (13)$$

$$H^*(w_x(z)) = S^*(w_x(z)) M_1(z) \quad (14)$$

$$IV_0(z) = \sum_{j=0}^{\infty} \alpha_0^j \left(\prod_{i=0}^j \beta_i \right) \frac{1 - VI^*(w_x(z))}{w_x(z)} \quad (15)$$

$$IVR(z) = A^*(\lambda) \frac{Q(z)}{\lambda} + M_1(z) IV_0(z) \quad (16)$$

$$Q(z) = \frac{-w_x(z)}{z - H^*(w_x(z))} IVR(z) \quad (17)$$

$$Y(z) = S^*(w_x(z)) Q(z) - I_0(z) \quad (18)$$

Thus the probability generating function during idle period is given by,

$$P_{idle}(z) = QI^*(z,0) + PI^*(z,0) + P_{I_0} = P_0(0) \frac{Q(z)}{w_x(z)} [S^*(w_x(z)) - z] \quad (19)$$

The total probability generating function $P(z)$ of the queue size probabilities is

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$$P(z) = P^*(z,0) + QI^*(z,0) + PI^*(z,0) + PI_0 = P_0(0) Q(z) \frac{1-z}{w_x(z)} \quad (20)$$

III. PERFORMANCE MEASURES

In this section some useful performance measures such as queue size probabilities and mean queue size when the system is in different states are calculated for the proposed model. The following results obtained from (12) to (18) are used to derive these measures.

$$I_0(1) = \phi ; \quad I_0'(1) = -\lambda E(X) \sum_{j=0}^{\infty} \alpha_0^j \left(\prod_{i=0}^j \beta_i \right) E(VI)$$

$$M_1(1) = 1 ; \quad M_1'(1) = E(X) (1 - A^*(\lambda)) ; \quad M_1''(1) = E(X(X-1)) (1 - A^*(\lambda))$$

$$[H^*(w_x(z))] |_{z=1} = 1 ; \quad [H^*(w_x(z))]' |_{z=1} = \lambda E(X) \left[E(S) + \frac{1-A^*(\lambda)}{\lambda} \right] = \rho ;$$

$$[H^*(w_x(z))]' |_{z=1} = \lambda E(X(X-1)) E(H) + (\lambda E(X))^2 E(H^2)$$

$$\text{where } E(H) = E(S) + \frac{1-A^*(\lambda)}{\lambda} ; \quad E(H^2) = E(S^2) + 2 E(S) \frac{1-A^*(\lambda)}{\lambda}$$

$$IV_0(1) = \sum_{j=0}^{\infty} \alpha_0^j \left(\prod_{i=0}^j \beta_i \right) E(VI) ; \quad IV_0'(1) = \frac{\lambda}{2} E(X) \sum_{j=0}^{\infty} \alpha_0^j \left(\prod_{i=0}^j \beta_i \right) E(VI^2)$$

$$IVR(1) = A^*(\lambda) \frac{\phi}{\lambda} + \sum_{j=0}^{\infty} \alpha_0^j \left(\prod_{i=0}^j \beta_i \right) E(VI) ;$$

$$IVR'(1) = E(X) \sum_{j=0}^{\infty} \alpha_0^j \left(\prod_{i=0}^j \beta_i \right) [(1 - A^*(\lambda)) E(VI) + \frac{\lambda}{2} E(VI^2)]$$

$$Q(1) = \frac{\lambda E(X)}{1-\rho} IVR(1) ; \quad Q'(1) = \frac{\lambda}{1-\rho} [IVR'(1) E(X) + \frac{IVR(1)}{2} (E(X(X-1))) + \frac{E(X)}{1-\rho} [H^*(w_x(z))]' |_{z=1}]$$

$$Y(1) = Q(1) - \phi ; \quad Y'(1) = Q'(1) + \lambda E(X) E(S) Q(1) - I_0'(1)$$

3.1 The Steady State Queue Size Probabilities

Let P_{VI} , P_I and P_{busy} denote the probability that the server is on idle vacation, idle state and busy state respectively. Then the corresponding probabilities can be obtained by using the above results in the equations (8) to (10).

- (i) $P_{VI} = \lim_{z \rightarrow 1} QI^*(z,0) = P_0(0) \sum_{j=0}^{\infty} \alpha_0^j \left(\prod_{i=0}^j \beta_i \right) E(VI)$
- (ii) $P_I = \lim_{z \rightarrow 1} PI^*(z,0) = P_0(0) Y(1) \frac{1-A^*(\lambda)}{\lambda}$
- (iii) $P_{busy} = \lim_{z \rightarrow 1} P^*(z,0) = P_0(0) Q(1) E(S)$
- (iv) $PI_0 = \frac{\phi}{\lambda} P_0(0)$

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$P_0(0)$ is calculated using the normalizing condition $P(1)=1$, and given by: $P_0(0) = \frac{\lambda E(X)}{Q(1)}$.

3.2 Mean Queue Size

Let L_{VI} , L_{idle} and L_{busy} denote the expected queue size when the server is in idle vacation, idle state and busy state respectively. Then the mean queue sizes are the derivatives of the equations (8) to (10) at $z=1$.

$$(i) L_{VI} = \frac{d}{dz} QI^*(z,0) |_{z=1} = P_0(0) \frac{\lambda}{2} E(X) E(VI^2) \sum_{j=0}^{\infty} \alpha_0^j (\prod_{i=0}^j \beta_i)$$

$$(ii) L_I = \frac{d}{dz} PI^*(z,0) |_{z=1} = P_0(0) Y'(1) \frac{1-A^*(\lambda)}{\lambda}$$

$$(iii) L_{busy} = \frac{d}{dz} P^*(z,0) |_{z=1} = P_0(0) [Q'(1) E(S) + \frac{\lambda}{2} E(X) E(S^2) Q(1)]$$

Thus the total expected queue size for the proposed model can be calculated as follows:

$$L = L_{VI} + L_I + L_{busy} = P_0(0) \left[\frac{IVR(1)}{1-\rho} \left[\frac{IVR'(1)}{IVR(1)} + \frac{\lambda E(X(X-1))E(H) + (\lambda E(X))^2 E(H^2)}{2(1-\rho)} \right] \right] \quad (21)$$

IV. NUMERICAL ANALYSIS

In this section, the queue size probabilities and the mean queue lengths are calculated corresponding to different parameters and different distribution of non-Markovian random variables. The distribution of each random variable and their measures used for the numerical computations of both the models are listed in the following table:

Random Variable (Y)	Distribution	Mean E(Y)	Second order moments E(Y ²)
Retrial Time (A)	Exponential (v_1)	$\frac{1}{v_1}$	$\frac{2}{v_1^2}$
Idle Vacation Time (VI)	Erlang 8-type ($8, \eta_1$) $\eta_1 = 0.2$	$\frac{1}{\eta_1}$	$\frac{9}{8\eta_1^2}$
Busy Vacation Time (VB)	Gamma 2-type ($2, \eta$) $\eta = 3$	$\frac{2}{\eta}$	$\frac{6}{\eta^2}$
Repair Time (R)	Exponential (rI) $rI = 4$	$\frac{1}{rI}$	$\frac{2}{rI^2}$

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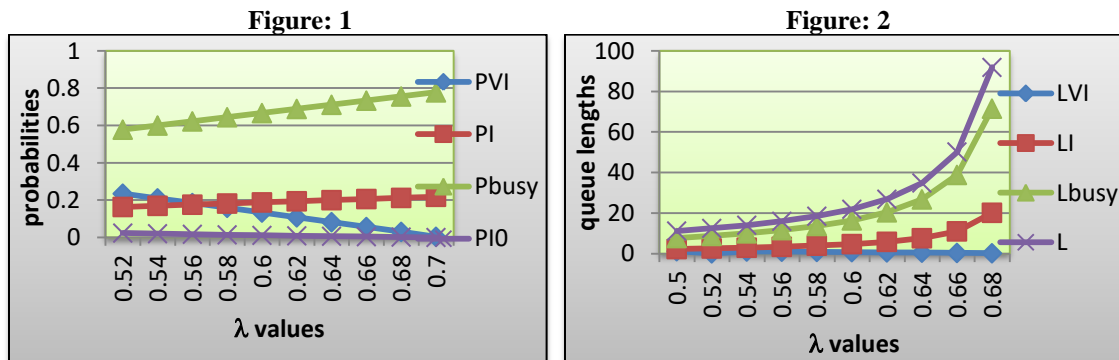
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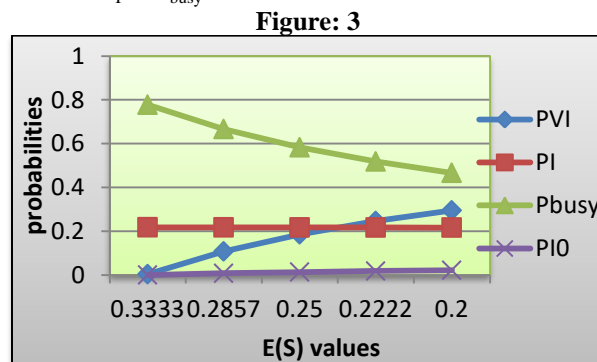
Batch Size (X)	Geometric (p_1) $p_1 = 0.7$	$\frac{1}{1 - p_1}$	$\frac{2 p_1}{(1 - p_1)^2}$
Service Time (S)	Erlang 5-type ($5, \mu_1$) $\mu_1 = 3$	$\frac{1}{\mu}$	$\frac{6}{5\mu^2}$

Figures 1,2 give the queue size probabilities and mean system size when the system is in different states respectively for the model. The following observations are made. As the group arrival rate (λ) increases,

- (i) queue size probability when the system is empty (PI_0) and in idle vacation state (P_{VI}) along the mean queue length during idle vacation period (L_{VI}) decrease and
- (ii) queue size probability during busy vacation (P_{VB}), breakdown (P_{br}), idle (P_I) and busy (P_{busy}) states and their corresponding mean queue lengths L_{VB} , L_{br} , L_I and L_{busy} and with the total mean queue length (L) increase.



The effects of the mean service time $E(S)$ are highlighted in the figure 3 for the model. The observation made is, as $E(S)$ decreases, P_{VI} & PI_0 increase and P_I & P_{busy} decrease.



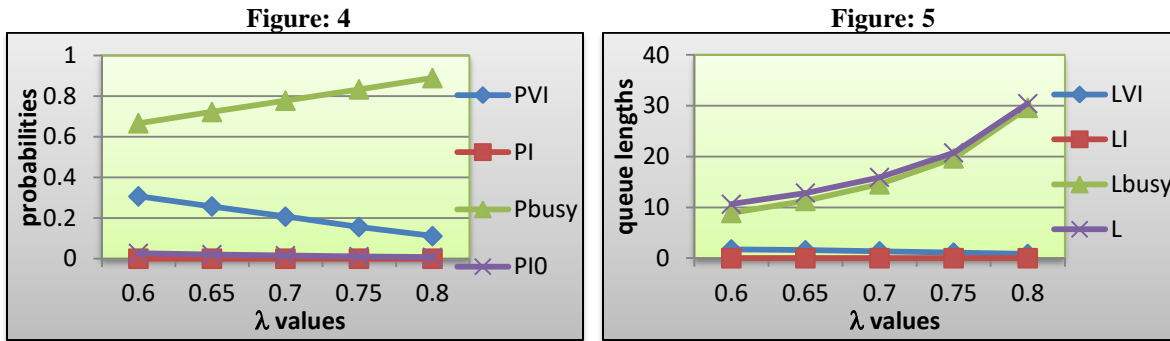
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It is shown in figures 4&5 that, as $v_1 \rightarrow \infty$, $A^*(\lambda) \rightarrow 1$ so that the retrial model results coincide with that of the corresponding classical queueing model.



V. PARTICULAR CASES

5.1 Decomposition Property

The total probability generating function $P(z)$ can be rewritten as

$$P(z) = \frac{(z-1)(1-\rho_{br})}{z-S^*(w_x(z))} \frac{P_{idle}(z)}{P_{idle}(1)} = \frac{(z-1)(1-\rho)}{z-H^*(w_x(z))} \frac{IVR(z)}{IVR(1)} \quad (22)$$

The equation (22) shows that, the probability generating function of the queue size of the model under consideration is decomposed into the product of two probability generating functions, one of which is the probability generating function of number of customers in the queue of $M^X/G/1$ queueing system without vacation and the other is the probability generating function of the conditional queue size distribution $\frac{P_{idle}(z)}{P_{idle}(1)}$ during the server idle period.

5.2 Total probability generating function for different vacation policies

In this section, the total probability generating function of the steady state queue size probabilities for the models under different vacation policies are deduced with the suitable selection of β_j 's. The decomposition property (equation (22)) shows that, the total probability generating function corresponding to different vacation policies differs from one another only in the expression of $IVR(z)$. The selection of β_j 's and corresponding $IVR(z)$ are given below:

Single Vacation Model ($\beta_0 = 1, \beta_j = 0 \forall j \geq 1$):

$$IVR(z) = A^*(\lambda) \frac{\alpha_0}{\lambda} + M_1(z) \frac{1-VI^*(w_x(z))}{w_x(z)}; \quad IVR(1) = A^*(\lambda) \frac{\alpha_0}{\lambda} + E(VI)$$

Multiple Vacation Model ($\beta_j = 1 \forall j \geq 0$):

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$$IVR(z) = \frac{M_1(z) \frac{1-VI^*(w_x(z))}{w_x(z)}}{1-\alpha_0}; \quad IVR(1) = \frac{1}{1-\alpha_0} E(VI)$$

Non-Vacation Model ($\beta_j = 0 \forall j \geq 0$):

$$IVR(z) = IVR(1) = \frac{A^*(\lambda)}{\lambda}$$

J-Vacation Model ($\beta_0 = 1, \beta_j = \bar{p}$ (a constant) $\forall 1 \leq j \leq J-1, \beta_j = 0 \forall j \geq J$):

$$IVR(z) = \frac{A^*(\lambda)}{\lambda} (\alpha_0(1-\bar{p}) \frac{1-(\alpha_0\bar{p})^{J-1}}{1-\alpha_0\bar{p}} + \alpha_0^J \bar{p}^{J-1}) + M_1(z) \frac{1-(\alpha_0\bar{p})^J \frac{1-VI^*(w_x(z))}{w_x(z)}}{1-\alpha_0\bar{p}}$$

$$IVR(1) = \frac{A^*(\lambda)}{\lambda} (\alpha_0(1-\bar{p}) \frac{1-(\alpha_0\bar{p})^{J-1}}{1-\alpha_0\bar{p}} + \alpha_0^J \bar{p}^{J-1}) + \frac{1-(\alpha_0\bar{p})^J}{1-\alpha_0\bar{p}} E(VI)$$

5.3 Classical $M^X/G/1$ queueing model

It is shown that the PGF of the queue size $P(z)$ of the **Classical $M^X/G/1$** model can also be reduced to corresponding classical model under the condition $A^*(\lambda) \rightarrow 1$ and given by:

$$P(z) = P_0(0) \frac{z-1}{z-S^*(w_x(z))} \left[\frac{\rho}{\lambda} + \sum_{j=0}^{\infty} \alpha_0^j \left(\prod_{i=0}^j \beta_i \right) \frac{1-VI^*(w_x(z))}{w_x(z)} \right]$$

VI. CONCLUSION

The main objective of the present work is to analyse the retrial bulk arrival queueing model under **Multiple Adapted Vacation policy** and obtain the steady state probabilities and mean queue lengths when the system is in different states. The decomposition property of vacation queues is established and the numerical computations for the performance measures are calculated. The steady state results for the model corresponding to different vacation policies including non-vacation retrial and **Classical $M^X/G/1$** are also derived.

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