



e-ISSN: 2319-8753 | p-ISSN: 2320-6710

IJRSET

International Journal of Innovative Research in
SCIENCE | ENGINEERING | TECHNOLOGY

INTERNATIONAL JOURNAL OF INNOVATIVE RESEARCH

IN SCIENCE | ENGINEERING | TECHNOLOGY

Volume 9, Issue 11, November 2020

ISSN INTERNATIONAL
STANDARD
SERIAL
NUMBER
INDIA

Impact Factor: 7.512

9940 572 462

6381 907 438

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Graph Theoretic Approach to Nodal Analysis of Electrical Circuits

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ABSTRACT: Graph theory is a branch in Mathematics and its results have applications in many areas of the computing, social and natural sciences. Here we study about some objects (points) connected by a particular relation among themselves. Graph theoretic models are used in various fields such as Physics, Chemistry, Computer Science, Communication Science, Electrical Engineering etc. It seems to be laborious to use the existing and traditional methods to find branch currents and loop currents of electrical circuits if it contains many branches. For this kind of difficulties graph theory is very useful and powerful technique. Using graph theory and matrix we can find the currents containing any number of branches. In this paper, we have analysed currents of electrical circuits using graph theory and our results are verified by nodal analysis of electrical circuits.

Keywords: Graph theory, Electrical circuits, Fundamental tie-set matrix, Branch impedance matrix.

2010 Mathematics subject classification: 05C50

I. INTRODUCTION

An electrical circuit is a collection of electrical components and devices. A graph representation of electrical circuit is represented in terms of points called vertices or nodes and lines called edges or branches. We apply the concept of graph theory to solve electrical circuit problems. The behaviour of electrical circuits generally depends upon the characteristics of each of internally connected elements and their connection rules. The connection rule gives a relation between electrical circuits with graph theory. A connected graph without closed path i.e. tree was implemented by Kirchhoff in 1847. He used graph theoretical concept in the calculation of currents in network or circuits. In 1892 this concept was improved by J.C.Maxwell [17]. The relation between Kirchhoff's methods for constructing the equations that describe an electric circuit and graphs was first clarified by Veblen in 1916. In this paper, we have developed a graph theoretic model for calculation of currents of electrical circuits containing 'n' branches by using topological form of Kirchhoff's Voltage Law.

II. REVIEW OF GRAPH THEORY IN ELECTRICAL CIRCUIT ANALYSIS

In recent years electronic simulation programs are used to determine the circuit equations by means of nodal analysis which shows a more direct way to describe the relation between graphs and electric circuits. In 1980, Satoru Fujishige developed an algorithm to solve graph realization problem. He used the fundamental circuit matrix or graphic matrix for developing it [18]. Istvan Fary in 1983 studied and developed some algorithms on generalised circuits [16]. In 1985, Karl Gustafson studied graph and networks using vector calculus and developed some results relating to divergence and curl [20]. In 1997, Gunther and Hoscheck adapted ROW method for simulation of electric circuits [19]. In 2005 R.B. Bapat [6] studied incidence matrix, Laplacian and distance matrix for a tree with attached graphs. They found some results for the minors of the incidence matrix and the Laplacian and for the inverse and the determinant of the distance matrix. In 2010, Giuseppe Ali proposed a model for integrated circuits using partial differential equation and algebra [3]. In 2010, Diestel [13] gave a topological approach for studying locally finite graphs that crucially incorporates their ends. Chang [11] in 2011 presented a model based on matrix in power supply networks. A review of fundamental and historic results of algebraic graph theory and electrical network analysis is given in [15].

III. FROM CIRCUIT TO GRAPH

A graph can be obtained from a circuit. We identify the graph $G = (V, E)$ where V is the set of vertices and E is the set of edges. The edge between i^{th} and j^{th} vertices can be denoted by $\{i, j\}$ ignoring the direction. Similarly the

notation (i,j) can be used for oriented edges, where 'i' is the start vertex and 'j' is the end vertex. In general, current source and voltage source are replaced by open circuit and short circuit respectively. It may be observed that a branch may contain more than one element. The choice of branches and nodes with respect to network element is flexible.

IV. MATRICES ASSOCIATED TO A GRAPH

Following are some examples of matrices obtained from graph.

a) Fundamental Tie Set Matrix (Fundamental Loop Matrix) $[B]$ [10]

This matrix is associated to a fundamental loop i.e. a loop formed by only one link (branch that does not belong to a particular tree) associated with other twigs (branches of tree). Here, all loops obtained from a particular tree form the rows and all branches form the columns such that

$$b_{ij} = \begin{cases} 1, & \text{if branch } b_j \text{ in loop } i \text{ is in the same direction as the loop} \\ -1, & \text{if branch } b_j \text{ in loop } i \text{ is in the opposite direction of the loop} \\ 0, & \text{if branch } b_j \text{ is not in loop } i \end{cases}$$

b) Branch Impedance Matrix $[Z_b]$ [10]

It is a square matrix of order 'm' where m is the number of branches having branch impedance as the diagonal elements and mutual impedance as off diagonal elements. If there is no transformer or mutual sharing then off diagonal entry is zero.

V. APPLICATION OF GRAPH THEORY IN NETWORK EQUILIBRIUM EQUATIONS [10]

If V_{SK} be the voltage source in a branch k having impedance z_k and carrying current i_k , then the branch voltage $v_k = z_k i_k + V_{SK}$

If we consider this equation for the whole circuit then matrix form of the above equation becomes $[V_b] = [Z_b][I_b] + [V_S]$ where $[Z_b]$ is the branch impedance matrix, $[I_b]$ is the column vector of branch currents and $[V_S]$ is the column vector of source voltage. Kirchhoff's Voltage Law states that the algebraic sum of voltages (or voltage drops) in any closed path of network that is traversed in a single direction is zero. Its matrix form is given by

$$\begin{aligned} [B][V_b] &= 0 \\ \Rightarrow [B]([Z_b][I_b] + [V_S]) &= 0 \\ \Rightarrow [B][Z_b][I_b] &= -[B][V_S] \end{aligned}$$

Also the branch current matrix equation is $[I_b] = [B^T][I_L]$, where $[I_L]$ is the loop current matrix and $[B^T]$ is the transpose of $[B]$.

$$\text{So, we get } [B][Z_b][B^T][I_L] = -[B][V_S]$$

VI. CIRCUIT WITH ONE UNKNOWN RESISTANCE AND ONE UNKNOWN VOLTAGE

In this case, we consider a circuit with some voltages and resistances where one resistance and one voltage are unknowns. Graph theoretic approach is applied to find the currents and then it will be verified by nodal analysis.

A) Graph theoretic approach:

Let us consider the following circuit

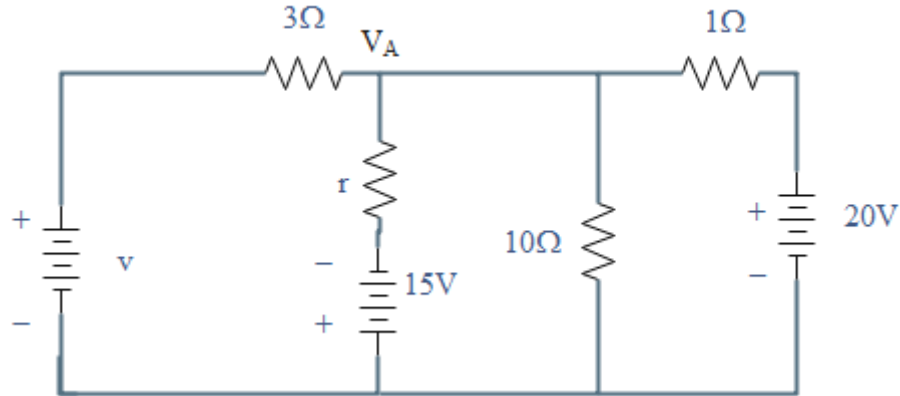


Figure 2.5: Circuit with one unknown resistance and voltage

Here,

$r_1 = 3\Omega$	$v_1 = v \text{ V (Volt)}$
$r_2 = r \Omega$	$v_2 = 15 \text{ V}$
$r_3 = 10\Omega$	$v_3 = 20 \text{ V}$
$r_4 = 1\Omega$	

The graph obtained from the circuit in Fig 2.5 is

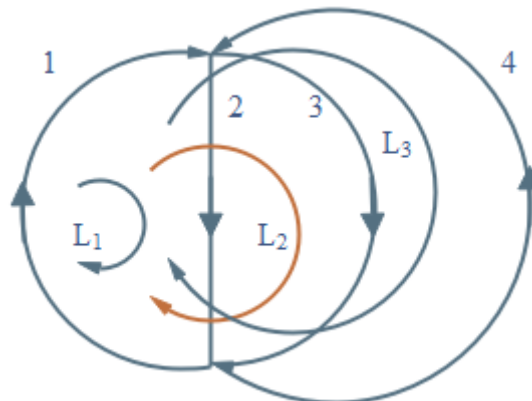


Figure 2.6: Graph from circuit

Here also branch 1 is considered as the tree branch to find the loops the circuit. From the graph we have following matrices



$$B = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & -1 \end{bmatrix}, \quad Z_b = \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & r & 0 & 0 \\ 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$B^T = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \quad I_L = \begin{bmatrix} I_{L_1} \\ I_{L_2} \\ I_{L_3} \end{bmatrix}, \quad V_s = \begin{bmatrix} v \\ 15 \\ 0 \\ 20 \end{bmatrix}$$

So putting in

$$[B][Z_b][B^T][I_L] = -[B][V_s]$$

$$\Rightarrow \begin{bmatrix} 3+r & 3 & 3 \\ 3 & 13 & 3 \\ 3 & 3 & 4 \end{bmatrix} \begin{bmatrix} I_{L_1} \\ I_{L_2} \\ I_{L_3} \end{bmatrix} = \begin{bmatrix} -(v+15) \\ -v \\ 20-v \end{bmatrix}$$

$$\Rightarrow X I_L = Y \text{ where } X = \begin{bmatrix} 3+r & 3 & 3 \\ 3 & 13 & 3 \\ 3 & 3 & 4 \end{bmatrix} \quad Y = \begin{bmatrix} -(v+15) \\ -v \\ 20-v \end{bmatrix}$$

Solving we have

$$I_{L_1} = \frac{-10v-1245}{43r+30} A$$

$$I_{L_2} = \frac{-60r-vr+45}{43r+30} A$$

$$I_{L_3} = \frac{260r-10vr+1050}{43r+30} A, \text{ Where } 43r+30 \neq 0$$

Again from

$$[I_b] = [B^T][I_L], \text{ the branch currents are given as}$$

$$\Rightarrow \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} \frac{200r-11vr-10v-150}{43r+30} \\ \frac{-10v-1245}{43r+30} \\ \frac{-60r-vr+45}{43r+30} \\ \frac{-260r+10vr-1050}{43r+30} \end{bmatrix}$$

B) Verification of the result by Nodal Analysis:

Now we will verify the above results using nodal analysis. We will find the branch currents. Let the nodal voltage at A be V_A and considering this as lead voltage from Kirchhoff's current law we have

$$\frac{V_A - v}{3} + \frac{V_A + 15}{r} + \frac{V_A}{10} + \frac{V_A - 20}{1} = 0$$

$$\Rightarrow V_A = \frac{600r + 10vr - 450}{43r + 30}$$

So the branch currents are given by

$$i_{3\Omega} = \frac{\frac{600r + 10vr - 450}{43r + 30} - v}{3} = \frac{200r - 11vr - 10v - 150}{43r + 30} A$$

$$i_{r\Omega} = \frac{\frac{600r + 10vr - 450}{43r + 30} + 15}{r} = \frac{1245 + 10v}{43r + 30} A$$

$$i_{10\Omega} = \frac{\frac{600r + 10vr - 450}{43r + 30}}{10} = \frac{60r + vr - 45}{43r + 30} A$$

$$i_{1\Omega} = \frac{\frac{600r + 10vr - 450}{43r + 30} - 20}{1} = \frac{-260r + 10vr - 1050}{43r + 30} A$$

So we see that in both processes the values of the currents are same. Negative sign is just showing that currents are in opposite direction.

Results: In $X I_L = Y$ we see that

- 1) The matrix X is symmetric
- 2) The diagonal elements of X
 - $a_{11} = 3 + r = \text{sum of resistances in } L_1$
 - $a_{22} = 3 + 10 = 13 = \text{sum of resistances in } L_2$
 - $a_{33} = 3 + 1 = 4 = \text{sum of resistances in } L_3$

So the a_{ii} element (diagonal) of matrix X is the sum of the resistances of loop L_i of I_L .

- 3) Without Using nodal analysis and mesh analysis we can find the loop current and branch current by using graph theory only.

VII. APPLICATION OF THE MODEL

Mmatrix method is very helpful to find the loop currents and branch currents of electrical circuits containing 'n' branches. It is laborious when we apply some traditional processes such as nodal analysis and some well known theorems of electrical circuits to find the branch currents containing many loops or branches. Our model will overcome this problem. In future it may be helpful to develop more complex circuit solving programs.

VIII. CONCLUSION

From the above study we observed that the result obtained by graph theoretic approach is same as the result obtained by Superposition Theorem. So the graph theoretic approach is verified by existing Superposition Theorem. The value of

currents can be obtained for different value of ' r_i ' and ' v_i ' by network equilibrium equation without knowing the nodal and mesh analysis of electrical circuits.

REFERENCES

- [1] Abeysinghe Sathara, Wu Jianzhong, Sooriyabandara Mahesh, A Statistical Assessment Tool for Electricity Distribution Networks, *Energy Procedia* 105 (2017), 2595 – 2600, Elsevier.
- [2] Akhmetbayev Dauren S., Aubakir Daurenbek A., Sarsikekey Yermek Zh., Bainiyazov Bakhtybek A., Surkov Mikhail A., Rozhkov Vitaliy I., Ansabekova Gulbahit N., Yerbolova Assel S., Suleimenov Azamat T., Tokasheva Miramgul S., Development of topological method for calculating current distribution coefficients in complex power networks, *Results in Physics* 7 (2017), 1644–1649, Elsevier.
- [3] Alì Giuseppe, PDAE models of integrated circuits, *Mathematical and Computer Modelling* 51 (2010), 915-926, Elsevier.
- [4] Alman Joshua, Lian Carl, Tran Brandon, Circular planar electrical networks: Posets and positivity, *Journal of Combinatorial Theory, Series A* 132 (2015), 58–101.
- [5] Banasode S. N., Umathar Y.M., Review of Applications of Graph Theory in Engineering, *International Journal of Mathematics Trends and Technology (IJMTT) - Volume 57, Issue 4- May 2018*.
- [6] Bapat R.B., Distance matrix and Laplacian of a tree with attached graphs, *Linear Algebra and its Applications* 411 (2005), 295–308, Elsevier.
- [7] Boruah Chanchal, Gogoi Krishna, Chutia Chandra. Analysis of Some Electrical Circuits with the Help of Graph Theory Using Network Equilibrium Equations, *International Journal of Innovative Research in Science, Engineering and Technology*, Vol. 6, Issue 1, January 2017.
- [8] Brtnik Bohumil, Matousek David, Compiling a Conductivity Matrix of a Switched Currents Circuits by Means of Two-Graphs, *Energy Procedia* 14 (2012), 167 – 172, Elsevier.
- [9] Burtsev Y.A., Simulation of electrical circuits using conjugate gradient algorithm, *Procedia Engineering* 129 (2015), 308 – 316, Elsevier.
- [10] Chakrabarti Abhijit, *Circuit Theory Analysis and Synthesis*, Dhanpat Rai & Co.
- [11] Chang Chen-Sung, A matrix-based VaR model for risk identification in power supply networks, *Applied Mathematical Modelling* 35 (2011), 4567–4574, Elsevier.
- [12] Chen Hai-Xiang, Yang Lei, Wang Ming-Jun, Electrical characteristics of n-ladder network with internal load, *Results in Physics* 15 (2019), 102488.
- [13] Diestel Reinhard, Locally finite graphs with ends: A topological approach, II. Applications, *Discrete Mathematics* 310 (2010), 2750-2765, Elsevier.
- [14] Dorfler Florian, Bullo Francesco, Kron Reduction of Graphs with Applications to Electrical Networks. *IEEE Transactions on Circuits and Systems I*, Volume: 60, Issue: 1, Jan. 2013.
- [15] Dorfler Florian, Simpson-Porco John W., Bullo Francesco, *Electrical Networks and Algebraic Graph Theory: Models, Properties, and Applications*, Vol. 106, No. 5, May 2018, Proceedings of the IEEE.
- [16] Fary Istvan, Generalised Circuits. Elements of Mathematical Theory, *Journal of Pure and Applied Algebra* 29 (1983), 241-283.
- [17] Fayez Mohamed, Graph Theory and its application in Electrical Power System, ResearchGate, 2016.
- [18] Fujishige Satoru, An Efficient PQ-Graph Algorithm for Solving the Graph-Realization Problem, *Journal of Computer and System Sciences* 21, 63-86 (1980).
- [19] Gunther M., Hoschek M., ROW methods adapted to electric circuit simulation packages, *Journal of Computational and Applied Mathematics* 82(1997), 159-170.
- [20] Gustafson Karl, Harary Frank, The Curl of Graphs and Networks, *Mathematical Modelling*, Vol. 6, pp 145-155, 1985.
- [21] Harary F., *Topics in Graph Theory*, New York Academy of Sciences, 1979.
- [22] Wang Shouxiang, Jiang Xingyue, Li Quinghui, Huang Bibin, Loop Analysis Method for Short Circuit Current Calculation of Distribution Network with Inverter-Interfaced Distributed Generators, *Energy Procedia*, 158(2019), 2909-2914.



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