



e-ISSN: 2319-8753 | p-ISSN: 2320-6710

IJRSET

International Journal of Innovative Research in
SCIENCE | ENGINEERING | TECHNOLOGY



INTERNATIONAL JOURNAL OF INNOVATIVE RESEARCH

IN SCIENCE | ENGINEERING | TECHNOLOGY

Volume 9, Issue 10, October 2020

ISSN INTERNATIONAL
STANDARD
SERIAL
NUMBER
INDIA

Impact Factor: 7.512

9940 572 462

6381 907 438

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QFPP: Solving Quadratic Fractional Programming Problem Using Modified Simplex Approach

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ABSTRACT: This paper presents Quadratic fractional objective programming problem (QFPP) with linear constraints, has been defined and developed. Problem was solved by using the modified simplex approach, by suggesting an algorithm for each method to solve the problem.

KEYWORDS: Quadratic fractional programming problems, feasible solution, objective function

I. INTRODUCTION

Quadratic fractional programming problems (i.e. ratio of objectives that have numerator and denominator) has attracted considerable research and interest since they have been utilized in production planning, financial and corporate planning, health care and hospital planning. Several methods to solve such problems are proposed in (1962) [2], their method depends on transforming this linear fractional programming problem (LFPP) to an equivalent linear program. Quadratic fractional programming problem (QFPP) in which objective function is the ratio of quadratic numerator and linear denominator with linear inequality constraints and non-negative decision variables. In 1965 KantiSwarup [11] proposed an algorithm to solve LFPP without converting it, into the linear form. S.D. Sharma [10] gave a simplex algorithm for Quadratic Programming if the objective function is in the form of product of two linear functions (both are positive) and subject to the constraints are linear inequalities. Here we are introducing an algorithm for solving QFPP. Methodology of this algorithm is based upon mainly KantiSwarup's fractional algorithm (KSFA) [11] and simplex method for quadratic programming [10].

II. PRELIMINARIES

Definition 2.1: The Mathematical form of QFPP is as follows:

$$\text{Max. } Z = \frac{(c_1^T X + \alpha)(c_2^T X + \beta)}{(d^T X + \gamma)}$$

Sub to:
 $AX \leq b$
 $X \geq 0$

Where,

- (i) A is an $m \times n$ matrix,
- (ii) b is an $m \times 1$ vector,
- (iii) c_1, c_2, d and X are $n \times 1$ vectors,
- (iv) c_1^T, c_2^T and d^T are transpose of c_1, c_2 and d respectively,
- (v) α and β are scalars,
- (vi) $d^T X + \beta > 0$ and
- (vii) $S = \{X: AX \leq b, X \geq 0\}$ is non empty and bounded.

Notations 2.2: Following notations are used to solve QFPP:

$$z^1 = c_{1B}X_B + \alpha, \quad z^2 = c_{2B}X_B + \beta, \quad z = d_B X_B + \gamma,$$

$$\Delta \xi_j^1 = c_{1B}X_j - c_{1j}, \quad \Delta \xi_j^2 = c_{2B}X_j - c_{2j}, \quad \Delta \xi_j = d_B X_j - d_j,$$



$$\mu_j = \left\{ \frac{X_B}{x_j}, x_j > 0 \right\} \text{ for non basic variables,}$$

$$\Delta_{1j} = z^2 \Delta \xi_j^1 + z^1 \Delta \xi_j^2 - \mu_j \Delta \xi_j^1 \Delta \xi_j^2,$$

$$\Delta_{2j} = \Delta \xi_j,$$

$$Z_1 = z^1 z^2,$$

$$Z_2 = z,$$

$$\Delta_j = Z_2 \Delta_{1j} - Z_1 \Delta_{2j},$$

$$Z = \frac{Z_1}{Z_2}.$$

III. ALGORITHM FOR SOLVING QFPP

Step1: First of all, write the standard form of the given QFPP.

Step2: Construct the initial simplex table by using the notations (2.2).

Step3: Compute $\Delta_j = Z_2 \Delta_{1j} - Z_1 \Delta_{2j}$ Two cases will arise:

If all $\Delta_j \geq 0$ then the optimal basic feasible solution is obtained.

If at least one $\Delta_j < 0$ then proceed on to the next step.

Step4: Choose

$$\Delta_r = \min. [\Delta_j, \Delta_j < 0]$$

then the corresponding variable of r^{th} will enter the basis

Step5: Find

$$\frac{x_{Bk}}{x_{kr}} = \min. \left\{ \frac{x_{Bi}}{x_{ir}}, x_{ir} > 0, i = 1, 2, \dots, m \right\}$$

then the variable x_k will leave the basis

Step6: Convert the key element (i.e. x_{kr}) into unity and all the other elements of the key column to zero by elementary row operations.

Step7: Go to step3 and repeat the process until we get an optimal basic feasible solution.

IV. NUMERICAL EXAMPLES

Example 4.1: Solve

$$\text{Max. } Z = \frac{(2x_1 + 3x_2)(4x_1 + 6x_2 + 1)}{3x_1 + 2x_2 + 2}$$

Sub. to:

$$\begin{aligned} -2x_1 + x_2 &\leq 1, & 2x_1 + x_2 &\leq 4 \\ x_1, x_2 &\geq 0 \end{aligned}$$

Solution: Standard form of the given LFPP is:

$$\text{Max. } Z = \frac{(2x_1 + 3x_2 + 0x_3 + 0x_4)(4x_1 + 6x_2 + 0x_3 + 0x_4 + 1)}{3x_1 + 2x_2 + 0x_3 + 0x_4 + 2}$$

Sub. to:

$$\begin{aligned} -2x_1 + x_2 + s_3 &= 1, & 2x_1 + x_2 + s_4 &= 8 \\ x_1, x_2, s_3, s_4 &\geq 0. \end{aligned}$$



Where s_3 and s_4 are slack variables. Initial table:

Table-4.1

$\alpha = 0$ $\beta = 1$ $\gamma = 2$				c_{1j}	2	3	0	0	
				c_{2j}	4	6	0	0	
				d_j	3	2	0	0	
B.V.	c_{1B}	c_{2B}	d_{1B}	X_B	x_1	x_2	s_3	s_4	Min. Ratio
s_3	0	0	0	1	-2	1	1	0	-
s_4	0	0	0	8	2	1	0	1	4
$z^1 = 0, \quad z^2 = 1$ $z = 2$ $\mu_1 = 4, \quad \mu_2 = 1$				$\Delta \xi_j^1$	-2	-3	0	0	
				$\Delta \xi_j^2$	-4	-6	0	0	
				$\Delta \xi_j$	-3	-2	0	0	
$Z_1 = 0$ $Z_2 = 2$				Δ_{1j}	-26	-21	-	-	
				Δ_{2j}	-3	-2	0	0	
$Z = Z_1/Z_2 = 0$				Δ_j	-52	-42	-	-	

Drop s_4 and introduce x_1

Table-4.2

$\alpha = 0$ $\beta = 1$ $\gamma = 2$				c_{1j}	2	3	0	0	
				c_{2j}	4	6	0	0	
				d_j	3	2	0	0	
B.V.	c_{1B}	c_{2B}	d_{1B}	X_B	x_1	x_2	s_3	s_4	Min. Ratio
s_3	0	0	0	9	0	2	1	1	9/2
x_1	2	4	3	4	1	1/2	0	1/2	8
$z^1 = 8, \quad z^2 = 17$ $z = 14$ $\mu_2 = 8, \quad \mu_4 = 9$				$\Delta \xi_j^1$	0	-2	0	1	
				$\Delta \xi_j^2$	0	-4	0	2	
				$\Delta \xi_j$	0	-1/2	0	3/2	
$Z_1 = 136$ $Z_2 = 14$				Δ_{1j}	-	-130	-	15	
				Δ_{2j}	0	-1/2	0	3/2	
$Z = Z_1/Z_2 = \frac{136}{14}$				Δ_j	-	-2206	-	243	



Drop x_2 and introduce s_3

Table-4.3

$\alpha = 0$ $\beta = 1$ $\gamma = 2$				c_{1j}	2	3	0	0	
				c_{2j}	4	6	0	0	
				d_j	3	2	0	0	
B.V.	c_{1B}	c_{2B}	d_{1B}	X_B	x_1	x_2	s_3	s_4	Min. Ratio
x_2	3	6	2	9/2	0	1	1/2	1/2	—
x_1	2	4	3	7/4	1	0	-1/2	1/4	—
$z^1 = 17, \quad z^2 = 35$ $z = 65/4$ $\mu_3 = 9, \quad \mu_4 = 7$				$\Delta \xi_j^1$	0	0	1/2	2	
				$\Delta \xi_j^2$	0	0	1	4	
				$\Delta \xi_j$	0	0	-1/2	7/4	
$Z_1 = 595$ $Z_2 = 65/4$				Δ_{1j}	—	—	30	82	
				Δ_{2j}	0	0	-1/2	7/4	
$Z = Z_1/Z_2 = \frac{476}{13}$				Δ_j	—	—	785	$\frac{1165}{4}$	

Hence the basic feasible solution of the given problem is:

$$Max. Z = \frac{476}{13} = 36.61, \quad x_1 = \frac{7}{4}, \quad x_2 = \frac{9}{2}$$

V. CONCLUSION

The numbers of application of LFPP are very large and it is not possible to give a comprehensive survey of all of them. However, an efficient method for the solution of general LFPP is still. This technique is useful to apply on numerical problems, reduces the labour work and save valuable time.

In this paper, we introduced a method to solve a linear fractional programming problem with interval coefficients in the objective function. In the proposed method, on using convex combination of the first and the last points of intervals instead of intervals and also using variable transformation, the initial problem is transformed into a nonlinear programming problem which finally is changed into a linear programming problem which has two more constraints and one more variable compare to the initial problem.

The method is designed in such a way that each and every points of the interval examined for achieving the best possible solution for the problem we gave an iterative procedure for solving linear fractional programming problems. Our procedure is based on the conjugate projection method for solving nonlinear programming problems with linear constraints.

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