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Mathematical Modeling of Dissolved Oxygen with Soluble BOD in a stream

Dr. Nitash Kaushik¹, Dr. Jitendra Pratap Singh², Dr.(Prof.) Babita Tyagi³

Professor, Greater Noida Institute of Technology, Greater Noida, India¹

Assistant Professor, Noida Institute of Engineering and Technology, Greater Noida, India²

Professor, Galgotias University, Greater Noida, India³

ABSTRACT: In the present work an attempt is made to develop an unsteady state model that is capable of predicting the biochemical oxygen demand (BOD) in soluble form and dissolved oxygen (DO) in a stream. A finite difference scheme is used to solve the differential equations representing BOD-DO balance in stream.

I. INTRODUCTION

A steady state model is one in which there are no variations with time of the variables being modeled (e.g., BOD and DO) nor of any model parameters e.g., the stream velocity or waste discharge rate. An unsteady state model, on the other hand, allows for variations with time. Unsteady state models are more complex and difficult to solve but they are evidently more realistic and comprehensive. A steady state model may sometimes be used, even though in water resources problems very few processes are time invariant. Consider a model of the DO distribution in a stream that has several industrial plants and combined sewer overflows discharging into it. During certain seasons, if the plants are operating at a constant rate of production and if there are no rain storms, the steady state DO models might be appropriate. However, if it suddenly rains and the combined sewers overflow into the stream, or if the plants begin to operate irregularly, the steady state model cannot handle the resulting situation.

The flow of water in streams, canals, reservoirs, lakes, pools, and free- surface flow in storm water drains, conduits, pipes, galleries, tunnels and culverts, in which the velocities change with time, is defined as unsteady flow. Flow in natural channel is always unsteady

O'Connell and Thomas (1965) considered the assumption of steady-state conditions as the major fault of the Streeter-Phelps equation and therefore used

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} = p - r + K_2 D - K_1 L \quad (1)$$

They solved equation (1) using the assumption that the p and r factors were constant with respect to time and position. Hence the solution is applicable only at a point for a short interval of flow time.

O' Connor (1967) wrote a general DO balance for a one-dimensional stream in which the oxygen demand of the waste is divided into the requirements for the carbonaceous material and those for the nitrogenous material. The basic equation of the model is as follows:

$$\frac{\partial C}{\partial t} + \frac{F}{A_x} \frac{\partial C}{\partial x} = K_2 (C_s - C) + p_{x,t} + K_1 L - K_N N_x - r_{x,t} - SE_{x,t} \quad (2)$$

where,

F = The volumetric rate of flow.

A_x = The cross-sectional area of flow at longitudinal position x.

K_N = The rate constant for the oxidation of nitrogenous material.

N_x = The concentration of nitrogenous material at x.



SE = The rate of removal of BOD by sedimentation.

p = The rate of production of C by photosynthesis.

r = The rate of consumption of dissolved oxygen by plant respiration.

The x,t subscript indicates the longitudinal position x at time t.

Di Toro and O' Connor (1968) studied the effect of time varying discharge and mean velocity on the BOD and DO distributions. They found that the standard deviations of the resultant BOD and DO distributions were approximately equal to the coefficient of variation of the velocity multiplied by the BOD or DO distribution. But this is neither rational nor scientific as the settlement of particles cannot continue for an indefinite time calculated using the mean velocity.

Most DO balances give one value for the DO concentration at a downstream point. Similarly most DO quality standards do not distinguish as to the degree or duration of a pollution violation. This consideration is important because a DO concentration that is sufficient on the average to assure healthy fish does not prevent fish kills.

II. PHYSICAL SYSTEM

The organic waste containing soluble BOD is being discharged into the stream. The area of cross-section of the stream is supposed to be constant with longitudinal direction. There is no other source of BOD in the study reach except at the outfall.

Consider a stream in which partially treated/untreated oxygen demanding waste is dumped into the stream through a point source (waste treatment plant). Let the source strength varies with time at the outfall itself which may be the case of malfunctioning of equipment or accidental spill of pollutant.

III.MATHEMATICAL MODELLING

Mathematical model is developed for the above stated system based on the following assumptions.

- Advection and dispersion are considered to be the two transport mechanism along with the decay of BOD.
- The effect of reaeration is same everywhere in stream.
- The temperature effect on decomposition rates on oxygen saturation is same everywhere in stream.
- The BOD in the pollutant is assumed to be decaying exponentially with time at the outfall itself.

The following differential equations representing BOD-DO balance in the streams

$$\frac{\partial B_d}{\partial t} + u(x) \frac{\partial B_d}{\partial x} = \frac{1}{A(x)} \frac{\partial}{\partial x} \left[D_L(x) A(x) \frac{\partial B_d}{\partial x} \right] - K_d B_d \quad (3)$$

$$\frac{\partial C}{\partial t} + u(x) \frac{\partial C}{\partial x} = \frac{1}{A(x)} \frac{\partial}{\partial x} \left[D_L(x) A(x) \frac{\partial C}{\partial x} \right] - K_d B_d + K_a (C' - C) \quad x > 0, t > 0$$

(4)

where,

B_d = Dissolved Part of BOD (mg/L)

C = Concentration of dissolved oxygen (mg/L)

C' = Saturation level of dissolved oxygen (mg/L)

A = Area of cross-section of the stream (m^2)

$D_L(x)$ = Coefficient of dispersion (m^2/s)

$u(x)$ = Mean Cross-Sectional flow velocity (m/s)

d = Depth of the stream (m)

K_a = Coefficient of reaeration(s^{-1})

K_d = Decomposition rate of dissolved BOD (s^{-1})

Boundary conditions:

$$B_d(x, 0) = 0, \quad C(x, 0) = C' ; \quad x > 0$$

$$B_d(0, t) = B_{0-d}, \quad C(0, t) = C' ; \quad t > 0$$

$$B_d(x, t) \rightarrow 0, \quad C(x, t) \rightarrow C' \text{ as } x \rightarrow \infty$$

IV. METHOD OF SOLUTION

Substitute $D = C' - C$ in equation (4), then we have

$$\frac{\partial D}{\partial t} + u(x) \frac{\partial D}{\partial x} = \frac{1}{A(x)} \frac{\partial}{\partial x} \left[D_L(x) A(x) \frac{\partial D}{\partial x} \right] + K_d B_d - K_a D, \quad x > 0, t > 0 \tag{5}$$

Using the transformations

$$B_d = b \exp(-K_d t), \quad D = d \exp(-K_a t)$$

Transformed Boundary Conditions:

$$b(x, 0) = 0, \quad d(x, 0) = 0 ; \quad x > 0$$

$$b(0, t) = B_{0-d} \exp(K_d t), \quad d(0, t) = 0 ; \quad t > 0$$

$$b(x, t) \rightarrow 0, \quad d(x, t) \rightarrow 0 \text{ as } x \rightarrow \infty$$

Equation (5) gives

$$\frac{\partial d}{\partial t} + v(x) \frac{\partial d}{\partial x} = D_L(x) \frac{\partial^2 d}{\partial x^2} + K_d b \exp[(K_a - K_d)t] \tag{6}$$

where,



$$V(x) = u(x) - \frac{D_L(x)}{A(x)} \frac{\partial}{\partial x} A(x) - \frac{\partial}{\partial x} D_L(x)$$

Equation (6) is split into two parts to give

$$\frac{1}{v(x)} \frac{\partial d}{\partial t} + \frac{\partial d}{\partial x} = \frac{K_d b}{v(x)} \exp[(K_a - K_d)t] \tag{7a}$$

And

$$\frac{\partial d}{\partial t} = D_L(x) \frac{\partial^2 d}{\partial x^2} \tag{7b}$$

The numerical solution of equation (7a) is obtained by discretizing it over a non-uniform rectangular grid as follows:

$$\begin{aligned} & \frac{1}{2} \left[\frac{d_{i+1,n+1} - d_{i,n+1}}{\delta x_i} + \frac{d_{i+1,n} - d_{i,n}}{\delta x_i} \right] + \frac{1}{v_i} \left[\theta \left\{ \frac{d_{i,n+1} - d_{i,n}}{\delta t} \right\} + (1 - \theta) \left\{ \frac{d_{i+1,n+1} - d_{i+1,n}}{\delta t} \right\} \right] \\ & = \frac{K_d b_{i,n}}{v_i} \exp\{(K_a - K_d)n \delta t\} \end{aligned} \tag{8}$$

Using $\delta x_i = v_i \delta t$

$$\begin{aligned} & \frac{1}{2} [d_{i+1,n+1} - d_{i,n+1} + d_{i+1,n} - d_{i,n}] + \theta \{d_{i,n+1} - d_{i,n}\} + (1 - \theta) \{d_{i+1,n+1} - d_{i+1,n}\} \\ & = K_d b_{i,n} \delta t \exp\{(K_a - K_d)n \delta t\} \end{aligned}$$

Or,

$$d_{i+1,n+1} = \frac{(1+2\theta)}{(3-2\theta)} d_{i,n} + \frac{(1-2\theta)}{(3-2\theta)} d_{i,n+1} + \frac{(1-2\theta)}{(3-2\theta)} d_{i+1,n} + \frac{2K_d \delta t \exp\{(K_a - K_d)n \delta t\}}{(3-2\theta)} b_{i,n} \tag{9}$$

Rearrangement of equation (9), gives the unconditional stable explicit finite difference scheme as

$$d_{i+1,n+1} = A_1 d_{i,n} + A_2 d_{i,n+1} + A_3 d_{i+1,n} + A_4 b_{i,n} \tag{10}$$

Using a similar procedure for equation(3), we get

$$b_{i+1,n+1} = A_1 b_{i,n} + A_2 b_{i,n+1} + A_3 b_{i+1,n} \tag{11}$$

where



$$A_1 = \frac{(1 + 2\theta)}{(3 - 2\theta)}$$

$$A_2 = A_3 = \frac{(1 - 2\theta)}{(3 - 2\theta)}$$

$$A_4 = \frac{2K_d \delta t \exp\{(K_a - K_d)n \cdot \delta t\}}{(3 - 2\theta)}$$

Expansion of $d_{i+1,n+1}$, $d_{i,n+1}$, $d_{i+1,n}$ etc. by means of Taylor's theorem in terms of $d_{i,n}$ upto second degree terms and their substitution in Eq.(11) gives,

$$\frac{\partial d}{\partial t} + v(x) \frac{\partial d}{\partial x} = K_d \delta t \exp[(K_a - K_d) n \delta t] b_{i,n} + \left[\frac{(1-2\theta)v^2 \delta t}{2} \right] \frac{\partial^2 d}{\partial x^2} + \dots(12)$$

Comparison of Equation (12) with Equation (6) suggests the third non-zero term on the right hand side of Equation (12) is a numerical dispersion error. It is observed that the parameter θ representing the weighted average can be chosen in such a way that the coefficient of the third term on the RHS of Eq.(12) represents the physical dispersion coefficient D_L in a particular grid, i.e.,

$$D_L = \left[\frac{(1-2\theta)v^2 \delta t}{2} \right] \tag{13}$$

$$\text{So, } \theta = 0.5 - \frac{D_L}{v^2 \delta t} = 0.5 - \frac{1}{p\#} ; 0 \leq \theta \leq 1$$

V. RESULTS AND DISCUSSIONS

The finite difference scheme is used to predict the BOD and DO concentration downstream of the source.

Following theoretical but rational data is used:

The area of the stream cross-section is assumed as $A= 200 \text{ m}^2$ and $Q=200 \text{ m}^3/\text{s}$ is assumed to be constant throughout the study reach. The stream velocity $u(x)$ (m/s) is computed as $u= Q/A$.

The coefficient of dispersion $D_L(x)$ is determined by the formula developed by Fischer for natural streams, $D_L(x) = [0.01 u^2 w^2 / (u^* d)]$, in which d is depth (m) of the stream assumed to be constant ($= 4 \text{ m}$), w is the width (m) as $w = 25 \text{ m}$ and u^* is the friction velocity which is assumed as 0.09 m/s . The K_d and K_a values are assumed to be $0.3 \times 10^{-4}/\text{s}$ and $0.6 \times 10^{-4}/\text{s}$ respectively and $B_{0-d} = 28.0 \text{ mg/L}$. The DO in the stream at upstream of the wastewater input point is assumed to be at a saturation level of 9.17 mg/L .

The variation in d and b values with time is computed by a repeated application of the schemes given by equations (10) and (11) respectively, with boundary conditions given by (5.30) and (5.31) for a segment. Such evaluated values were taken as the input data for the next grid point, and this process was continuously repeated till the concentration in the last segment of the considered reach is computed.

The computed values of $b_{i+1,n+1}$, and $d_{i+1,n+1}$, were transformed into the $B_{i+1,n+1}$, and $D_{i+1,n+1}$ values through the use of the transformation used earlier. Finally, the DO concentration $C_{i+1,n+1}$ is computed using $C_{i+1,n+1} = C' - D_{i+1,n+1}$. The concentration of BOD and DO at the different distances were computed and are used to prepare BOD and DO v/s time plots at five different distances : 4 Km, 9 Km, 14 Km, 19 Km and 24 Km along the stream stretch.

Figure 1 shows the comparison of BOD values with time in the main zone at 4 Km, 9 Km, 14 Km, 19 Km and 24 Km distances respectively. The graph shows that the pollutant take some time to reach a particular distance and till that time the BOD concentration is negligible. Afterwards, the advection, dispersion and decay of pollutant take place

simultaneously and the pollutant start demanding oxygen from stream's DO. In the starting BOD concentration remains constant for a period at particular distances. When the mixing is over, the exponential decay will continue. The concentration of remaining BOD is decreasing with distance downstream and various curves would merge.

Figure 2 depicts the comparison of DO values with time in the main zone at 4 Km, 9 Km, 14 Km, 19 Km and 24 Km distances respectively. The pattern of the graph shows that the pollutant takes some time to reach a particular distance and till that time the BOD concentration is negligible and DO is at saturation level. Afterwards, the advection, dispersion and decay of pollutant take place simultaneously and the pollutant starts demanding oxygen from stream's DO. It is observed that, DO concentration is decreasing uniformly upto critical-point. Then DO is increasing up to the point of inflection and after that reaeration is dominant and DO deficit recover and attains the saturation level.

VI. CONCLUSION

It is observed that the concentration of remaining BOD is decreasing with time at particular distances. The reverse is happening in case of DO. This reflects that while moving along with the flow, the BOD is also decaying, a smaller amount of BOD is remaining and hence consuming a smaller amount of DO to fulfill the demand for oxygen, with increasing distance. Engineers and planners have interest in the lowest point, i.e., critical point of DO-sag curve. The concentration of critical DO decreases with increasing distance and it takes more time to attain critical deficit condition with distance downstream.

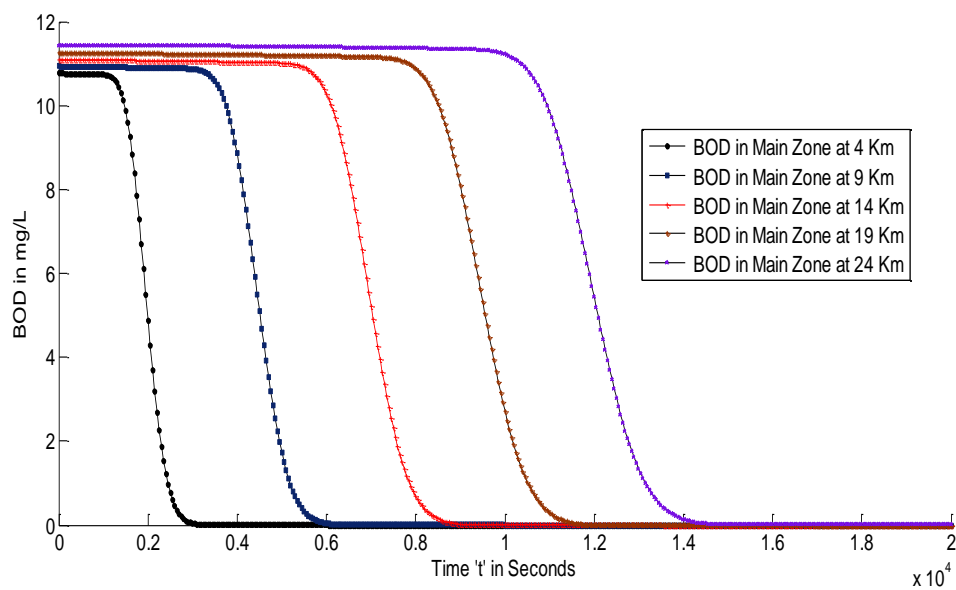


Figure 1: BOD distribution with soluble BOD under unsteady state conditions at different distances

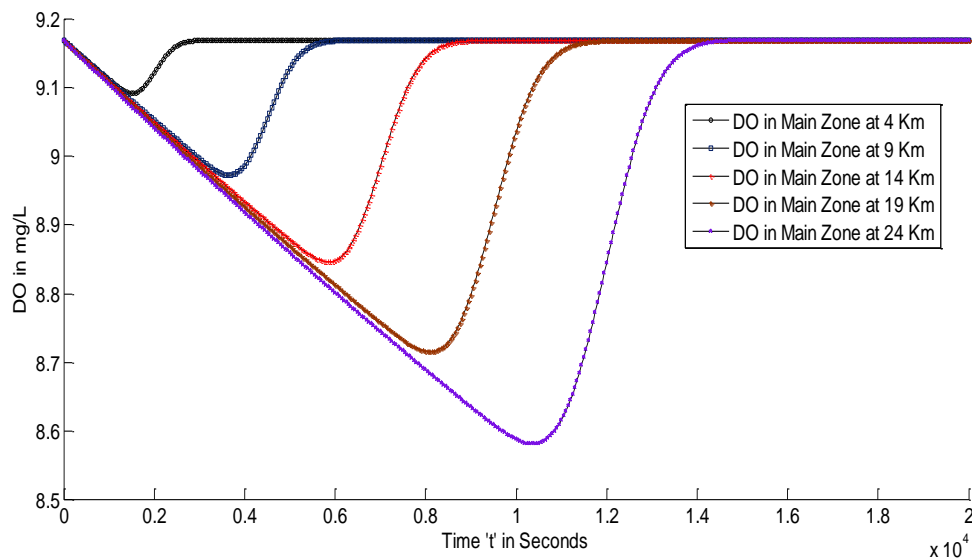


Figure 2: DO distribution with soluble BOD under unsteady state conditions at different distances

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