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A*- Closed Sets and I*-Closed Sets in Topology

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ABSTRACT: The purpose of this paper is to define and study new class of sets, called the class of A*-closed sets and I*-closed sets using α -closure operator, semiopen sets and gsg-open sets. Also, the basic properties of these new g-closed sets are obtained.

Mathematics Subject Classification : 54A05, 54B05, 54C08, 54D10

KEYWORDS and Phrases : α -open sets, semiopen sets, gsg-open sets, g*-closed sets, sg-open sets and semi- α -open sets, semi- α -closed sets.

I. INTRODUCTION

After, introduction of g-closed sets in topology for the first time by Levine [15], it is noticed that many of the weaker forms of closed sets have been generalized in topological spaces. In 1963, 1965, 1982 and 1986, respectively, N. Levine [14], O. Njastad [23], A.S. Mashhour et al [18] and D. Andrijevic [1] have defined and studied the concepts of semiopen sets, α -open sets, preopen sets and semipreopen sets in topology. In 1993, 1994, 1995, 1998, 2000, 2002, 2006 and 2011, respectively, H. Maki et al [16&17], Dontchev [8], Noiri et al [24], M.K.R.S. Veera Kumar [32&36], P. Rajrajeswari et al [27], and M.L. Thivagar et al [31] have defined and studied the concepts of $g\alpha$ -closed sets, αg -closed sets, gsp-closed sets, gp-closed sets, g*-closed sets, g#s-closed sets, strongly αg -closed sets and gsg-closed sets. The purpose of this paper is to define and study new class of generalized closed sets, called the class of A*-closed sets, A**[#]-closed sets, A[#]-closed sets and I*-closed sets using α -closure operator, semiopen sets, gs-open sets, sg-open sets and gsg-open sets. Also, the basic properties of these A*-closed sets, A*-open sets and I*-closed sets were obtained in this paper.

II. PRELIMINARIES

Throughout this paper (X, τ) and (Y, σ) (or simply X and Y) always means topological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a subset of space X. We denote the closure of A and the interior of A by $Cl(A)$ and $Int(A)$ respectively. A subset A of a space X is called regular open (in brief, r-open) if $A = Int(Cl(A))$ and regular closed (in brief, r-closed) if $A = Cl(Int(A))$.

The following definitions and results are useful in the sequel :

Definition 2.1 : A subset A of a space X is said to be:

- (i) preopen [18] if $A \subset Int(Cl(A))$.
- (ii) semiopen [14] if $A \subset Cl(Int(A))$.
- (iii) semipreopen [1] if $A \subset Cl(Int(Cl(A)))$.
- (iv) α -open [23] if $A \subset Int(Cl(Int(A)))$.
- (v) α -open [22] if $A \subset Cl(Int(Cl(Int(A)))$.

The complement of a preopen (resp. semiopen, semipreopen, α -open, α -open) set of a space X is called preclosed [9] (resp. semiclosed [4&6], semipreclosed [1], α -closed [20], α -closed [25]).

Definition 2.2 [1]: The union of all semipreopen sets contained in A is called the semipreinterior of A and is denoted by $spInt(A)$.

$pInt(A)$ [19] and $sInt(A)$ [7], $\alpha Int(A)$ [23] can be similarly defined.



Definition 2.3[1]: The intersection of all semipreclosed sets containing A is called the semipreclosure of A and is denoted by $spCl(A)$.

$pCl(A)$ [9] and $sCl(A)$ [6] , $\alpha Cl(A)$ [23] can be similarly defined .

Definition 2.4 : A subset A of a space X is called :

- (i) g-closed) set [15] if $Cl(A) \subset U$ whenever $A \subset U$ and U is open in X .
- (ii) gs-closed set [2] if $sCl(A) \subset U$ whenever $A \subset U$ and U is open in X .
- (iii) sg-closed) set [4] if $sCl(A) \subset U$ whenever $A \subset U$ and U is semiopen in X .
- (iv) gsp-closed) set [8] if $spCl(A) \subset U$ whenever $A \subset U$ and U is open in X .
- (v) gp-closed) set [24] if $pCl(A) \subset U$ whenever $A \subset U$ and U is open in X .
- (vi) g^* -closed set [32] if $Cl(A) \subset U$ whenever $A \subset U$ and U is g-open in X .
- (vii) g^*sp -closed set [21] if $spCl(A) \subset U$ whenever $A \subset U$ and U is g-open in X .
- (viii) gr^* -closed set [12] if $rCl(A) \subset U$ whenever $A \subset U$ and U is g-open in X .
- (ix) $(gsp)^*$ -closed) set [11] if $Cl(A) \subset U$ whenever $A \subset U$ and U is gsp-open in X .
- (x) α^* -closed set [37] if $Cl(A) \subset U$ whenever $A \subset U$ and U is α - open in X .
- (xi) αg -closed set [17] if $\alpha Cl(A) \subset U$ whenever $A \subset U$ and U is open in X .
- (xii) $g\alpha$ -closed set [16] if $\alpha Cl(A) \subset U$ whenever $A \subset U$ and U is α -open in X .
- (xiii) αg^* -closed set [30] if $\alpha Cl(A) \subset U$ whenever $A \subset U$ and U is αg -open in X .
- (xiv) $g^\#$ -closed set [35] if $Cl(A) \subset U$ whenever $A \subset U$ and U is αg -open in X .
- (xv) $g^\#s$ -closed set [36] if $sCl(A) \subset U$ whenever $A \subset U$ and U is αg -open in X .
- (xvi) $(gs)^*$ -closed set [10] if $Cl(A) \subset U$ whenever $A \subset U$ and U is gs-open in X .
- (xvii) g^*s -closed set [22] if $sCl(A) \subset U$ whenever $A \subset U$ and U is gs-open set in X .
- (xviii) strongly αg -closed set[27] if $\alpha Cl(A) \subset U$ whenever $A \subset U$ and U is g-open set in X .
- (xix) gp^* -closed set [13] if $Cl(A) \subset U$ whenever $A \subset U$ and U is gp-open set in X .
- (xx) \hat{g} -closed set[33] if $Cl(A) \subset U$ whenever $A \subset U$ and U is semiopen set in X .
- (xxi) $*g$ -closed set [33] if $Cl(A) \subset U$ whenever $A \subset U$ and U is \hat{g} -open set in X .
- (xxii) gsg-closed [31] if $Cl(A) \subset U$, whenever $A \subset U$ and U is sg-open set in X .
- (xxiii) αgs -closed set [26] if $\alpha Cl(A) \subset U$ whenever $A \subset U$ and U is semiopen set in X .
- (xxiv) ψ -closed set [34] if $sCl(A) \subset U$ whenever $A \subset U$ and U is sg-open set in X .
- (xxv) ψ^* -closed set [38] if $sCl(A) \subset U$ whenever $A \subset U$ and U is ψ -open set in X .

The complement of a m-closed set of a space X is called m-open set in X where $m = g , gs, sg, gsp, gp, \dots$ etc., which are cited in Definition-2.4

Definition 2.5 : The union of all α -open sets contained in A is called the α -interior of A and is denoted by $\alpha\text{Int}(A)$.

Definition 2.6 : The intersection of all α -closed sets containing A is called the α -closure of A and is denoted by $\alpha\text{Cl}(A)$.

Lemma 2.7 : For every subset $A \subset X$, the following hold.

- (i) $\alpha\text{Cl}(X-A) = X-\alpha\text{Int}(A)$.
- (ii) $\alpha\text{Int}(X-A) = X-\alpha\text{Cl}(A)$.

Definition 2.8 : A function $f : X \rightarrow Y$ is called :

- (i) semicontinuous [14] if the inverse image of each open set of Y is semiopen in X.
- (ii) α -continuous [20] if the inverse image of each open set of Y is α -open in X.
- (iii) α -continuous [25] if the inverse image of each open set of Y is α -open in X.

Definition 2.9 [5]: A space X is said to be g-Hausdroff if whenever x and y are distinct points of X there exist disjoint g-open sets U and V such that $x \in U$ and $y \in V$.

III. PROPERTIES OF A^* -CLOSED SETS

We introduce the following .

Definition 3.1: A subset A of a space X is said to be A^* -closed set if $\alpha\text{Cl}(A) \subset U$ whenever $A \subset U$ and U is semiopen in X.

The complement of a A^* -closed set is called A^* -open set.

Lemma 3.2: Every closed set (and hence semiclosed set , α -closed set , α -closed set) is A^* -closed set.

Lemma 3.3: Every sg-closed set (and hence g^\wedge -closed set , ψ -closed set , (gsp)*-closed set , g^*s -closed set , α gs-closed set) is A^* -closed set.

Lemma 3.4: Every g-closed set (and hence gs-closed set , gsp-closed set) and A^* -closed set are independent each other.

Definition 3.5: A subset A of a space X is said to be A^{**} -closed set if $\alpha\text{Cl}(A) \subset U$ whenever $A \subset U$ and U is gs-open in X.

The complement of a A^{**} -closed set is called A^{**} -open set.

Clearly , every A^{**} -closed set $\rightarrow A^*$ -closed set.

Definition 3.6: A subset A of a space X is said to be $A^\#$ -closed set if $\alpha\text{Cl}(A) \subset U$ whenever $A \subset U$ and U is sg-open in X.

The complement of a $A^\#$ -closed set is called $A^\#$ -open set.

Clearly , every A^{**} -closed set $\rightarrow A^\#$ -closed set and $A^\#$ -closed set $\rightarrow A^*$ -closed set. Thus we have , **every A^{**} -closed set $\rightarrow A^\#$ -closed set $\rightarrow A^*$ -closed set** .

Lemma 3.7: A subset A of space X is called A^* -open set if $F \subset \alpha\text{Int}(A)$ whenever $F \subset A$ and F is semiclosed set in X

We , define the following

Definition 3.8 : The union of all A^* -open sets which contained in A is called the A^* -interior of A and is denoted by $a^*\text{Int}(A)$.

Definition 3.9 : The intersection of all A^* -closed sets containing set A is called the A^* -closure of A and is denoted $a^*\text{Cl}(A)$.

Lemma 3.10 : Let $x \in X$, then $x \in a^*\text{Cl}(A)$ if and only if $V \cap A \neq \emptyset$ for every A^* -open set V containing x.

We , state the following.

Lemma 3.11: Let A and B be subsets of X. Then

- (i) $a^*\text{Cl}(\emptyset) = \emptyset$ and $a^*\text{Cl}(X) = X$
- (ii) If $A \subset B$, $a^*\text{Cl}(A) \subset a^*\text{Cl}(B)$.
- (iii) $A \subset a^*\text{Cl}(A)$

Lemma 3.12: Let A and B be subsets of X. Then

- (i) $a^*\text{Cl}(A) = a^*\text{Cl}(a^*\text{Cl}(A))$
- (ii) $a^*\text{Cl}(A) \cup a^*\text{Cl}(B) \subset a^*\text{Cl}(A \cup B)$



(iii) $a^*Cl(A \cap B) \subset a^*Cl(A) \cap a^*Cl(B)$

Theorem 3.13: If $s\alpha Int(A) \subset B \subset A$ and A is A^* -open, then B is A^* -open.

Proof: Obvious.

We, define the following.

Definition 3.14 : Let $x \in X$. A subset U of X is called a a^* -neighbourhood (in brief, a^* -nbd.) of x in X if there exists a A^* -open set B such that $x \in B \subset U$.

Note 3.15 : Let B be a A^* -open subset of a space X . Then by definition of a^* -nbd of $x \in X$, it is clear that B is a a^* -nbd. of each of its points, since for every $x \in B$, $x \in B \subset B$ and B is A^* -open set in X .

Definition 3.16 : Let $B \subset X$ and p be a point of X . Then p is called a limit point of B if for every a^* -nbd. U of p , we have $U \cap (B - \{p\}) \neq \emptyset$, or equivalently, $p \in a^*Cl(B - \{p\})$.

The set of all limit points of B is called the a^* -derived set of B and is denoted by $a^*D(B)$.

The following result is clear from the definition .

Lemma 3.17 : Let $B \subset X$. B is A^* -closed if and only if it contains the set of its a^* -limit points.

We, prove the following.

Lemma 3.18 : For any set B of a space X , $a^*Cl(B) = B \cup a^*D(B)$.

Proof : Let $p \in a^*D(B)$. Then, $p \in a^*Cl(B - \{p\}) \subset a^*Cl(B)$. Hence $a^*D(B) \subset a^*Cl(B)$. Also, $B \subset a^*Cl(B)$. Thus, we have $B \cup a^*D(B) \subset a^*Cl(B)$. Next, let $p \in a^*Cl(B)$. If $p \in B$, then $p \in B \cup a^*D(B)$. If $p \notin B$, then $B = B - \{p\}$. Hence, $p \in a^*Cl(B - \{p\})$. This implies that $p \in a^*D(B)$. Hence, $a^*Cl(B) \subset B \cup a^*D(B)$. So, we get $a^*Cl(B) = B \cup a^*D(B)$.

We, introduce the following.

Definition 3.19 : A space X is said to be a :

- (i) $aT_{1/2}^*$ -space if every A^* -closed set in it is closed .
- (ii) aT_{α}^* -space if every A^* -closed set in it is α -closed .
- (iii) aT_s -space if every A^* -closed set in it is semi-closed.

Definition 3.20: A function $f: X \rightarrow Y$ is called a^* -continuous if $f^{-1}(V)$ is A^* -closed in X for every closed subset V of Y .

Clearly, every continuous (and hence semicontinuous, α -continuous and $s\alpha$ -continuous) function is a^* -continuous.

Definition 3.21 : A function $f: X \rightarrow Y$ is called a^* -irresolute continuous if $f^{-1}(V)$ is A^* -closed in X for every A^* -closed subset V of Y .

Clearly, every a^* -irresolute function is a^* -continuous.

We, prove the following

Theorem 3.22 : Let $f : X \rightarrow Y$ be a function, then the following are equivalent :

- (i) f is a^* -continuous.
- (ii) The inverse image of each open set A in Y is A^* -open in X .

Proof : Easy.

Theorem 3.23 : Let $f : X \rightarrow Y$ be a^* -continuous and $g : Y \rightarrow Z$ be continuous, then $g \circ f : X \rightarrow Z$ is a^* -continuous.

Proof : Easy.

Theorem 3.24 : Let $f : X \rightarrow Y$ be a^* -irresolute and $g : Y \rightarrow Z$ be a^* -continuous, then $g \circ f : X \rightarrow Z$ is a^* -continuous.

Proof : Easy.

Theorem 3.25 : If $f : X \rightarrow Y$ be a^* -continuous with X as $aT_{1/2}^*$ -space, then f is continuous.

Theorem 3.26 : If $f : X \rightarrow Y$ be a^* -continuous with X as aT_{α}^* -space, then f is α -continuous.

Theorem 3.27 : If $f : X \rightarrow Y$ be a^* -continuous with X as aT_s -space, then f is semicontinuous.

Theorem 3.28 : A function $f : X \rightarrow Y$ is a^* -irresolute if for each A^* -open set A of Y , $f^{-1}(A)$ is A^* -open set in X .

Proof : Easy.

We, define the following.

Definition 3.29 : A function $f : X \rightarrow Y$ is said to be a^* -closed if the image of each closed set of X is A^* -closed set in Y .

Clearly, every closed function is a^* -closed function.

We, define the following.

Definition 3.30: A space X is called a^*-T_0 if to each pair of distinct points x, y of X , there exists a A^* -open set containing x but not y or, there exists a A^* -open set containing y but not x .

Theorem 3.31 : For a space X , the following are equivalent .

- X is a a^*-T_0 space ,
- For any two distinct points $x, y \in X$, $a^*Cl\{x\} \neq a^*Cl\{y\}$,
- For any two points $x, y \in X$, $a^*Cl\{x\} = a^*Cl\{y\} \Rightarrow x = y$.

Proof : (i) \Rightarrow (ii) . Let $x, y \in X$ with $x \neq y$. Suppose there is a A^* -open set U in X containing x not y . Then , $U \cap \{y\} = \emptyset$. This implies that $U \cap a^*Cl\{y\} = \emptyset$. Hence , $x \in a^*Cl\{x\}$ and $x \notin a^*Cl\{y\}$. Therefore , $a^*Cl\{x\} \neq a^*Cl\{y\}$.

(ii) \Rightarrow (iii). Easy.

(iii) \Rightarrow (i). Let $x, y \in X$ with $x \neq y$. Then from (iii) , $a^*Cl\{x\} \neq a^*Cl\{y\}$. Suppose , there is a point $p \in a^*Cl\{x\}$ and $p \notin a^*Cl\{y\}$. Then , $X - a^*Cl\{y\}$ is an A^* -open set containing p . Hence , $X - a^*Cl\{y\} \cap \{x\} \neq \emptyset$. Hence , $x \in X - a^*Cl\{y\}$ and $X - a^*Cl\{y\}$ is an A^* -open set containing x and not y . So, X is a^*-T_0 space.

We, define the following.

Definition 3.32 : A function $f : X \rightarrow Y$ is said to be point- a^* -closure 1-1 if and only if $x, y \in X$ such that $a^*Cl\{x\} \neq a^*Cl\{y\}$ then $a^*Cl\{f(x)\} \neq a^*Cl\{f(y)\}$.

Lemma 3.33 : If $f : X \rightarrow Y$ is point- a^* -closure 1-1 and X is a a^*-T_0 , then f is 1-1.

Proof follows from Th.3.29 and Def.-3.30.

Lemma 3.34 : Let $f : X \rightarrow Y$ be a function from a a^*-T_0 space X into a a^*-T_0 space Y . Then , f is point- a^* -closure 1-1 if and only if f is 1-1.

Proof follows from Lemma -3.31 above.

We, prove the following

Theorem 3.35 : Let $f : X \rightarrow Y$ is an a^* -irresolute injective function. If Y is a a^*-T_0 then X is a a^*-T_0 .

Proof : Let $x, y \in X$ with $x \neq y$. Since f is an injective and Y is a a^*-T_0 space , there exists a A^* -open set V_x in Y such that $f(x) \in V_x$ and $f(y) \notin V_x$, or there exists an A^* -open set V_y in Y such that $f(y) \in V_y$ and $f(x) \notin V_y$ with $f(x) \neq f(y)$. By a^* -irresoluteness of f , $f^{-1}(V_x)$ is an A^* -open set in X such that $x \in f^{-1}(V_x)$ and $y \notin f^{-1}(V_x)$ or $f^{-1}(V_y)$ is an A^* -open set in X such that $y \in f^{-1}(V_y)$ and $x \notin f^{-1}(V_y)$. This shows that X is a a^*-T_0 space.

We, define the following.

Definition 3.36 : A space X is called a^*-T_1 if to each pair of distinct points x, y of X , there exists a pair of A^* -open sets, one containing x but not y , and other containing y but not x .

In view of Definitions and results of $a^*Cl(A)$, $a^*-nbd.$ and $a^*D(A)$, one can prove the following.

Lemma 3.37 : A space X is a a^*-T_1 if and only if for any point $x \in X$, the singleton set $\{x\}$ is a A^* -closed set in X .

Lemma 3.38 : Let X be a a^*-T_1 space and $A \subset X$ and $p \in X$. Then , $a^*D(A)$ is a A^* -closed set of X .

Now, we state the following.

Theorem 3.39 : Let $f : X \rightarrow Y$ is an a^* -irresolute injective function. If Y is a a^*-T_1 then X is a a^*-T_1 .

Proof is similar to Th.3.33 above.

We, prove the following.

Theorem 3.40 : If $f : X \rightarrow Y$ is a a^* -closed surjective function and X is T_1 -space, then Y is a a^*-T_1 .

Proof : Let $y \in Y$ and $x \in X$ such that $f(x) = y$. As $\{x\}$ is a closed set in X since X is T_1 -space, $f(\{x\}) = \{y\}$ is a A^* -closed set in Y . Hence, Y is a a^*-T_1 space.

Next , we define the following .

Definition 3.41 : A function $f : X \rightarrow Y$ is said to be pre- a^* -closed if the image of each A^* -closed set of X is A^* -closed set in Y .

So, we state the following.

Theorem 3.42 : If $f : X \rightarrow Y$ is a pre- a^* -closed surjective function and X is a a^*-T_1 -space, then Y is a a^*-T_1 .

Proof is similar to Th.3.38 above.

We, define the following.

Definition 3.43: A space X is said to be a^* -Hausdorff if whenever x and y are distinct points of X , there exist disjoint A^* -open sets U and V such that $x \in U$ and $y \in V$.

Definition 3.44 :A function $f : X \rightarrow Y$ is called g a^* -continuous if the inverse image of each g -open set of Y is A^* -open in X .

We, prove the following.

Theorem 3.45: Let X be a space and Y be Hausdorff. If $f: X \rightarrow Y$ be a^* -continuous injective, then X is a^* -Hausdorff.

Proof : Let x and y be any two distinct points of X . Then $f(x)$ and $f(y)$ are distinct points of Y , because f is injective. Since Y is Hausdorff, there exist two disjoint open sets U and V such that $f(x) \in U$ and $f(y) \in V$. Since f is a^* -continuous and $U \cap V = \emptyset$, we have $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint A^* -open sets in X such that $x \in f^{-1}(U)$ and $y \in f^{-1}(V)$. Hence X is a^* -Hausdorff space.

Theorem 3.46 : Let X be a space and Y be a^* -Hausdorff. If $f: X \rightarrow Y$ be a^* -irresolute injective, then X is a^* -Hausdorff.

Proof : Similar to Th.3.23 above.

We state the following.

Theorem 3.47 : Let X be a space and Y be g -Hausdorff. If $f: X \rightarrow Y$ be g a^* -continuous injective, then X is a^* -Hausdorff.

IV. PROPERTIES OF I^* -CLOSED SETS

We introduce the following.

Definition 4.1: A subset A of a space X is said to be I^* -closed set if $\alpha Cl(A) \subset U$ whenever $A \subset U$ and U is gsg -open in X .

The complement of a I^* -closed set is called I^* -open set.

In view of Definitions-2.4, we have the following implications hold true :

Lemma 4.2 : Every, closed (and hence α -closed, semiclosed, $\alpha\alpha$ -closed and r -closed) set is I^* -closed set.

Lemma 4.3: Every, I^* -closed set is gsp -closed set.

Lemma 4.4 : Every, ψ -closed (and hence g^* -closed, $g\#$ -closed, $g\#s$ -closed, strongly αg -closed, gsg -closed, ψ^* -closed, g^*s -closed, $(gs)^*$ -closed, $(gsp)^*$ -closed, αg^* -closed, gr^* -closed, gp^* -closed) set is I^* -closed set.

Lemma 4.5 : Every, semipreclosed set (and hence g^\wedge -closed set, g^*sp -closed set, αg -closed set, $g\alpha$ -closed set, sg -closed set, gs -closed set, g -closed set and g^* -closed set) and I^* -closed set are independent..

We, define the following

Definition 4.6 : The union of all I^* -open sets which contained in A is called the I^* -interior of A and is denoted by $i^*Int(A)$.

Definition 4.7 : The intersection of all I^* -closed sets containing set A is called the I^* -closure of A and is denoted $i^*Cl(A)$.

Lemma 4.8 : Let $x \in X$, then $x \in i^*Cl(A)$ if and only if $V \cap A \neq \emptyset$ for every I^* -open set V containing x .

Remark 4.9 : The $i^*Cl(A)$ is I^* -closed set.

We, state the following.

Lemma 4.10: Let A and B be subsets of X . Then

- (i) $i^*Cl(\emptyset) = \emptyset$ and $i^*Cl(X) = X$
- (ii) If $A \subset B$, $i^*Cl(A) \subset i^*Cl(B)$.
- (iii) $A \subset i^*Cl(A)$

Lemma 4.11: Let A and B be subsets of X . Then

- (i) $i^*Cl(A) = i^*Cl(i^*Cl(A))$
- (ii) $i^*Cl(A) \cup i^*Cl(B) \subset i^*Cl(A \cup B)$
- (iii) $i^*Cl(A \cap B) \subset i^*Cl(A) \cap i^*Cl(B)$

We, define the following.

Definition 4.12: A function $f: X \rightarrow Y$ is called i^* -continuous if $f^{-1}(V)$ is I^* -closed in X for every closed subset V of Y .

Definition 4.13 : A function $f: X \rightarrow Y$ is called i^* -irresolute continuous if $f^{-1}(V)$ is I^* -closed in X for every I^* -closed subset V of Y .



Definition 4.14: A function $f : X \rightarrow Y$ is called strongly i^* -continuous if the inverse image of each I^* -open set of Y is open in X .

We, prove the following

Theorem 4.15 : Let $f : X \rightarrow Y$ be a function, then the following are equivalent :

- (iii) f is i^* -continuous.
- (iv) The inverse image of each open set A in Y is I^* -open in X .

Proof : Easy.

Theorem 4.16: Let $f : X \rightarrow Y$ be i^* -continuous and $g : Y \rightarrow Z$ be continuous , then $g \circ f : X \rightarrow Z$ is i^* -continuous.

Proof : Easy.

Theorem 4.17: A function $f : X \rightarrow Y$ is i^* -irresolute if for each I^* -open set A of Y , $f^{-1}(A)$ is I^* -open set in X .

Proof : Easy.

We define the following.

Definition 4.18 : A space X is said to be a :

- (iv) $iT_{1/2}^*$ - space if every I^* -closed set in it is closed .
- (v) iT_{α}^* -space if every I^* -closed set in it is α -closed .
- (vi) iT_{ψ} - space if every I^* -closed set in it is ψ -closed.
- (vii) $gspT_i$ -space if every gsp -closed set in it is I^* -closed.
- (viii) iT_{gsg} -space if every I^* -closed set in it is gsg -closed.

Lemma 4.19 : A subset A of space X is called I^* -open set if $F \subset s\alpha Int(A)$ whenever $F \subset A$ and F is gsg -closed set in X

Theorem 4.20: If $s\alpha Int(A) \subset B \subset A$ and A is I^* -open, then B is I^* -open.

Proof: Let A be I^* -open and $s\alpha Int(A) \subset B \subset A$. Then $X - A \subset X - B \subset X - s\alpha Int(A)$ that implies $X - A \subset X - B \subset s\alpha Cl(X - A)$. Thus , $X - B$ is I^* -closed. This proves that B is I^* -open

We, define the following.

Definition 4.21: A space X is said to be i^* -Hausdorff if whenever x and y are distinct points of X , there exist disjoint I^* -open sets U and V such that $x \in U$ and $y \in V$.

Definition 4.22: A space X is said to be s -Hausdorff if whenever x and y are distinct points of X there exist disjoint semi-open sets U and V such that $x \in U$ and $y \in V$.

Definition 4.23: A space X is said to be α -Hausdorff if whenever x and y are distinct points of X there exist disjoint α -open sets U and V such that $x \in U$ and $y \in V$.

Definition 4.24 : A function $f : X \rightarrow Y$ is called g_i^* -continuous if the inverse image of each g -open set of Y is I^* -open in X .

Definition 4.25 : A function $f : X \rightarrow Y$ is called s_i^* -continuous if the inverse image of each semiopen set of Y is I^* -open in X .

Definition 4.26 : A function $f : X \rightarrow Y$ is called α_i^* -continuous if the inverse image of each semiopen set of Y is I^* -open in X .

Next , we prove the following.

Theorem 4.27: Let X be a space and Y be Hausdorff . If $f : X \rightarrow Y$ be i^* -continuous injective , then X is i^* -Hausdorff .

Proof : Let x and y be any two distinct points of X . Then $f(x)$ and $f(y)$ are distinct points of Y ,

because f is injective. Since Y is Hausdorff, there exist two disjoint open sets U and V such that $f(x) \in U$ and $f(y) \in V$. Since f is i^* -continuous and $U \cap V = \emptyset$, we have $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint I^* -open sets in X such that $x \in f^{-1}(U)$ and $y \in f^{-1}(V)$. Hence X is i^* -Hausdorff space.

Theorem 4.28 : Let X be a space and Y be i^* -Hausdorff. If $f: X \rightarrow Y$ be i^* -irresolute injective, then X is i^* -Hausdorff.

Proof : Similar to Th.3.23 above.

We state the following.

Theorem 4.29 : Let X be a space and Y be g -Hausdorff. If $f: X \rightarrow Y$ be g - i^* -continuous injective, then X is i^* -Hausdorff.

Theorem 4.30 : Let X be a space and Y be s -Hausdorff. If $f: X \rightarrow Y$ be s - i^* -continuous injective, then X is i^* -Hausdorff.

Theorem 4.31: Let X be a space and Y be α -Hausdorff. If $f: X \rightarrow Y$ be α - i^* -continuous injective, then X is i^* -Hausdorff.

We define the following.

Definition 4.32: A function $f: X \rightarrow Y$ is said to be i^* -open if the image of each open set of X is I^* -open in Y .

Definition 4.33: A function $f: X \rightarrow Y$ is said to be always i^* -open if the image of each I^* -open set of X is I^* -open in Y .

Clearly, every always i^* -open function is i^* -open.

Definition 4.34 : A function $f: X \rightarrow Y$ is said to be i^* -closed if the image of each closed set of X is I^* -closed in Y .

We prove the following.

Lemma 4.35 : A surjective function $f: X \rightarrow Y$ is i^* -closed iff for each subset B of Y and each open set U of X containing $f^{-1}(B)$, there exists a I^* -open set V of Y such that $B \subset V$ and $f^{-1}(V) \subset U$.

Proof : Suppose f is i^* -closed function. Let B be any subset of Y and U be an open set of X containing $f^{-1}(B)$. Put $V = Y - f(X - U)$. Then V is I^* -open set in Y , $B \subset V$ and $f^{-1}(V) \subset U$.

Conversely, Let F be any closed set of X . Put $B = Y - f(F)$, then we have $f^{-1}(B) \subset X - F$ and $X - F$ is open set in X . There exists a I^* -open set in Y such that $B = Y - f(F) \subset V$. Therefore, we obtain $f(F) = Y - V$ and hence $f(F)$ is I^* -closed set in Y . Thus, f is i^* -closed function.

Definition 4.36: A function $f: X \rightarrow Y$ is said to be always i^* -closed if the image of each I^* -closed set of X is I^* -closed in Y .

Clearly, every always i^* -closed function is i^* -closed.

We, prove the following.

Theorem 4.37: Let $f: X \rightarrow Y$ be a bijection. Then the following are equivalent.

- (i) f is always i^* -open.
- (ii) f is always i^* -closed.
- (iii) f^{-1} is i^* -irresolute.

Proof: Obvious

We, define the following.

Definition 4.38: A function $f: X \rightarrow Y$ is said to be strongly i^* -open if the image of each I^* -open set of X is open in Y .

Theorem 4.39: Let $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ be any two functions. Then,

- (i) If f is i^* -open and g is always i^* -open, then $g \circ f$ is i^* -open.
- (ii) If f is strongly i^* -open and g is i^* -open, then $g \circ f$ is always i^* -open.
- (iii) If f is always i^* -open and g is strongly i^* -open, then $g \circ f$ is strongly i^* -open.



Proof : Easy and hence omitted.
We , define the following .

Definition 4.40: A function $f:X \rightarrow Y$ is said to be strongly i^* -closed if the image of each I^* -closed set of X is closed in Y .

Theorem 4.41: Let $f :X \rightarrow Y$ and $g : Y \rightarrow Z$ be any two functions. Then,

- (i) If f is i^* -closed and g is always i^* -closed , then $g \circ f$ is i^* -closed.
- (ii) If f is strongly i^* -closed and g is i^* -closed , then $g \circ f$ is always i^* -closed.
- (iii) If f is always i^* -closed and g is strongly i^* -closed ,then $g \circ f$ is strongly i^* -closed.

Proof : Easy proof is omitted.

Definition 4.42: A bijection $f:X \rightarrow Y$ is called i^* -homeomorphism if f is both i^* -continuous function and i^* -open function .

Theorem 4.43 : Every homeomorphism is an i^* -homeomorphism..
We , define the following.

Definition 4.44 : A bijection $f : X \rightarrow Y$ is said to be always i^* -homeomorphism if both f and f^{-1} are i^* -irresolute functions.

Theorem 4.45: Every always i^* -homeomorphism is an i^* -homeomorphism.

Proof: Obvious.

V.PROPERTIES OF I^* -REGULAR SPACES

We introduce the following.

Definition 5.1 : A space X is said to be i^* -regular if for each I^* -closed set F and each $x \in X-F$, there exist disjoint open sets U and V in X such that $x \in U$ and $F \subset V$.

Clearly, every i^* -regular space is regular space .

We, prove the following.

Theorem 5.2 : The following statements are equivalent for a space X :

- (i) X is i^* -regular space
- (ii) For each I^* -open set U containing $x \in X$, there exists an open set K in X such that $x \in K \subset Cl(K) \subset U$.

Proof : Easy .

Theorem 5.3 : A space X is i^* -regular if for each I^* -closed set F of X and each point $x \in X-F$, there exist open sets K and H of X such that $x \in K$, $F \subset H$ and $Cl(K) \cap Cl(H) = \emptyset$

Proof : Let F be a I^* -closed set of X and $x \in X-F$.There exist open sets G_0 and H of X such that $x \in G_0$, $F \subset H$ and $G_0 \cap H = \emptyset$; and hence $G_0 \cap Cl(H) = \emptyset$. Since X is i^* -regular space , there exist open sets of X such that $x \in U$, $Cl(H) \subset V$ and $U \cap V = \emptyset$; and hence $Cl(U) \cap V = \emptyset$. Now , put $K = G_0 \cap U$, then K and H are open sets of X such that $x \in K$, $F \subset H$ and $Cl(K) \cap Cl(H) = \emptyset$.

Theorem 5.4 : If $f : X \rightarrow Y$ be an open i^* -irresolute bijection and X is i^* -regular , then Y is i^* -regular .

Proof : Let F be any I^* -closed set of Y and $y \in Y-F$. Since f is i^* -irresolute and hence $f^{-1}(F)$ is I^* -closed set in X .Put $f(x) = y$, then $x \in X-f^{-1}(F)$, since X is i^* -regular , there exist disjoint open sets K and H such that $x \in K$ and $f^{-1}(F) \subset H$. As f is open bijective function , we obtain $y \in f(K)$, $F \subset f(H)$ and $f(K) \cap f(H) = \emptyset$. Thus , Y is i^* -regular .



We define the following

Definition 5.5: A space X is said to be i^* -regular if for each closed set F and each $x \in X - F$, there exist disjoint I^* -open sets U and V in X such that $x \in U$ and $F \subset V$.

Clearly, every i^* -regular space is i^* -regular space.

Next, we state the following for a regular spaces :

Theorem 5.6: The following properties are equivalent for a space X :

- (i) X is regular space;
- (ii) for each closed set F and each point $x \in X - F$, there exist disjoint gsg-open sets U and V of X such that $x \in U$ and $F \subset V$.
- (iii) for each closed set F and each point $x \in X - F$, there exist disjoint ψ -open sets U and V of X such that $x \in U$ and $F \subset V$.
- (iv) for each closed set F and each point $x \in X - F$, there exist disjoint I^* -open sets U and V of X such that $x \in U$ and $F \subset V$.
- (v) for each closed set F and each point $x \in X - F$, there exist disjoint gsp-open sets U and V of X such that $x \in U$ and $F \subset V$.
- (vi) for each $x \in X$ and an open set V containing x there is a gsp-open set K in X such that $x \in K \subset \text{spCl}(K) \subset V$.

Proof : (i) \rightarrow (ii) . Since every open set is gsg-open set.

(ii) \rightarrow (iii) . Since every gsg-open set is ψ -open set.

(iii) \rightarrow (iv) . Since every ψ -open set is I^* -open set .

(iv) \rightarrow (v) . Since every I^* -open set is gsp-open set.

(v) \rightarrow (vi) . Let $x \in X$ and V be an open set containing x . Then, $(X - V)$ is a closed set not containing x . Then by hypothesis, there are gsp-open sets K and H in X such that $x \in K, X - V \subset H$ and $K \cap H = \emptyset$. So, $x \in K \subset X - H \subset V$. As $X - V$ is closed, we have $X - V \subset \text{spInt}(H)$ and $K \cap \text{spInt}(H) = \emptyset$. Therefore, we obtain $\text{spCl}(K) \cap \text{spInt}(H) = \emptyset$ and hence $x \in K \subset (X - \text{spInt}(H)) \subset V$. This implies the required result.

(vi) \rightarrow (i) . Let $x \in X$ and A be a closed set not containing x . Then, $x \in X - A$ and $(X - A)$ be an open set. Then by hypothesis, there is a gsp-open set K such that $x \in K \subset \text{spCl}(K) \subset (X - A)$. Since A is closed set we have $A \subset \text{spInt}(K)$. Put $B = \text{Int Cl Int}(\text{spInt}(K))$ and $H = \text{Int Cl Int}(X - \text{spCl}(K))$. Then B and H are disjoint open sets such that $x \in B$ and $A \subset H$. This shows that space X is regular.

Next, we state the following.

Theorem 5.7: Let $f : X \rightarrow Y$ be a continuous i^* -closed surjection with compact point inverses. If X is regular then Y is i^* -regular .

The routine proof is easy by Lemma – 3.39.

Theorem 5.8: Let $f : X \rightarrow Y$ is an open i^* -irresolute bijection and X is i^* regular then Y is i^* -regular .

VI. CONCLUSION

In this paper, we defined and studied the concepts of A^* -closed sets, a^* -continuous functions, a^* -irresolute functions, a^* -separation axioms, I^* -closed sets, i^* -continuous functions, i^* -irresolute functions and i^* -regularity axioms. Still, there are some more properties of these sets can be established.

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