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Some Properties and Applications of Arctan Generalized Exponential Distribution

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ABSTRACT: A novel three-parameter model called arctan generalized exponential distribution is developed and studied using the generalized exponential distribution as parent distribution. Some of its useful properties are mathematically and statistically derived and analyzed. While estimating the model parameters, three well-known estimation methods are employed which are Cramer-Von-Mises (CVM), maximum likelihood estimation (MLE) and least-square estimation (LSE) methods. Two real sets of data are applied for illustrating the flexibility and suitability of the novel developed distribution. To assess the validity of the proposed distribution, AICC, AIC, HQIC and BIC are also calculated. All the computations are performed using R-software.

KEYWORDS: Arctan distribution, Generalized exponential, Quantile function, Survival function.

I. INTRODUCTION

Lifetime distributions are generally used to study life span of components of a system, a device, and in general, reliability and survival analysis. Lifespan distributions are commonly used in areas such as life science, medicine, biology, engineering, insurance, etc. Various continuous probability distributions for instance Weibull, Exponential and Gamma have been commonly used in statistical literature to examine lifespan data. For a few years, most of the researchers are attracted towards the exponential distribution for its likely in modeling lifespan data, and it has been perceived that this distribution has done outstandingly in several uses due to the existence of closed form solutions to many survival analyses. It can easily be justified under the supposition of constant failure rate but in the practice, the failure rates are not always constant. Therefore, haphazard use of exponential lifetime model seems to be inappropriate and unrealistic. In modern years, novel classes of models have been introduced centered on adjustments of the existing classical probability models, Marshall and Olkin (2007).

Newly, several efforts have been made to generate novel distributions to lengthen recognized distributions as well as to deliver unlimited flexibility for data encountered. Some procedures might be used to form a greater family by including additional parameters to an established distribution. So, some models with addition of extra parameters to produce novel models have been recommended in the statistical literature, Rinne (2009) and Pham and Lai (2007).

Let Y denote a random variable that follows exponential distribution having λ as parameter where CDF can be expressed as follows

$$F_Y(y; \lambda) = 1 - e^{-\lambda y} ; y > 0, \lambda > 0 \quad (1)$$

Many generalizations exponential distribution has been used in statistical literatures to generate more flexible life-time models. Some of them well known generalizations are as follows:

Smith and Bain (1975) were developed exponential power distribution by the modification of exponential distribution. Chaudhary and Kumar (2020) have introduced the logistic modified exponential distribution. Chaudhary and Kumar (2020) have also introduced a three-parameter complementary exponential power model for estimating model parameters using MCMC method. Gupta and Kundu (1999) were presented the generalized exponential distribution which is flexible than exponential distribution, having increasing and decreasing failure rate hazard function with PDF as

$$f_{GE}(x; \alpha, \lambda) = \alpha \lambda e^{-\lambda x} \{1 - e^{-\lambda x}\}^{\alpha-1} ; (\alpha, \lambda) > 0, x > 0. \quad (2)$$

The logistic-exponential distribution was given by Lan and Leemis (2008). Nadarajah and Haghghi (2011) generalized the exponential distribution and called it as an extension of exponential distribution. Its PDF can have unimodal and decreasing shapes, and the failure rate shows decreasing and increasing shapes. Joshi (2015) has proposed another extension of exponential distribution called new extended exponential (EEN) distribution having monotonically



increasing and constant hazard rate shapes. The continuous random variable X follows EEN with α and λ as parameters where it's CDF is

$$F(x) = 1 - \exp(-\alpha x e^{-\lambda/x}) ; x > 0, (\alpha, \lambda) > 0 \tag{3}$$

Chaudhary and Kumar (2020) have introduced the half logistic exponential extension distribution using exponential extension distribution as a base distribution. Joshi and Kumar (2020) have presented the new exponential extension Poisson distribution using exponential extension as baseline distribution. Chaudhary et al. (2020) have presented an extension of exponential distribution named truncated Cauchy power exponential distribution. The Lindley exponential power model was given by (Joshi and Kumar, 2020). Joshi et al. (2020) have introduced another distribution using exponential power as a parent distribution called the logistic-exponential power distribution. The truncated Cauchy power inverse exponential distribution was developed by (Chaudhary et al., 2020).

Gómez-Déniz & Calderín-Ojeda (2015) have presented the arctan distribution which was used for modeling insurance data of Norwegian fire. This family of distribution has been suggested for an primary Pareto distribution and the new distribution called Pareto arctan distribution and established that this distribution provide a good fit as compared to some well-known distributions. The CDF and PDF of arctan family of distribution with support [a, b] is

$$F(x) = 1 - \frac{\arctan[\alpha\{1-G(x)\}]}{\arctan(\alpha)} ; x \geq 0, \alpha > 0; \alpha \in [a,b] \tag{4}$$

$$f(x) = \frac{1}{\arctan(\alpha)} \frac{\alpha g(x)}{1 + [\alpha\{1-G(x)\}]^2} ; x \geq 0, \alpha > 0 \tag{5}$$

In this paper we aim to present a model with more flexibility while achieving a good fit to real dataset. We investigate the properties of the arctan generalized exponential distribution and illustrate its applicability. The new arctan generalized exponential distribution is presented and numerous distributional properties are given in Section 2. The model parameters are estimated using LSE, CVME and MLE methods in Section 3. In Section 4, two real sets of data are used to analyze and exploring the flexibility and suitability of the novel introduced distribution. In this segment, the parameters' ML estimators and approximate confidence intervals are also presented for the above-mentioned method of estimations. We also calculate AICC, AIC, BIC and HQIC to judge the legitimacy of the arctan generalized exponential model. Finally, Section 5 ends up through some overall concluding explanations.

II. THE ARCTAN GENERALIZED EXPONENTIAL (ATGE) DISTRIBUTION

The CDF and PDF of generalized exponential distribution (Gupta & Kundu, 1999)'s respectively can be written as

$$G(x) = (1 - e^{-\lambda x})^\beta ; x \geq 0, (\beta, \lambda) > 0 \tag{6}$$

$$g(x) = \lambda \beta e^{-\lambda x} (1 - e^{-\lambda x})^{\beta-1} ; x \geq 0, (\beta, \lambda) > 0 \tag{7}$$

The Arctan generalized exponential distribution's CDF and PDF with α, β and λ as parameters can be found by using the (4) and (5) where parent distributions are (6) and (7) and can be stated as

$$F(x) = 1 - \frac{\arctan[\alpha\{1-(1-e^{-\lambda x})^\beta\}]}{\arctan(\alpha)} ; x \geq 0, (\alpha, \beta, \lambda) > 0 \tag{8}$$

$$f(x) = \frac{\alpha \beta \lambda e^{-\lambda x} (1 - e^{-\lambda x})^{\beta-1}}{\arctan(\alpha) \{1 + [\alpha\{1 - (1 - e^{-\lambda x})^\beta\}]^2\}} ; x \geq 0, (\alpha, \beta, \lambda) > 0 \tag{9}$$

Survival function/Reliability function

The survival function of arctan generalized exponential (ATGE) distribution is

$$R(x) = 1 - F(x) = \frac{\arctan[\alpha\{1-(1-e^{-\lambda x})^\beta\}]}{\arctan(\alpha)} ; x \geq 0, (\alpha, \beta, \lambda) > 0 \tag{10}$$



Hazard function

The hazard rate function of ATGE distribution can be well-defined as,

$$h(x) = \frac{f(x)}{R(x)} = \frac{\alpha\beta\lambda e^{-\lambda x} (1 - e^{-\lambda x})^{\beta-1}}{\arctan[\alpha\{1 - (1 - e^{-\lambda x})^\beta\}]\{1 + [\alpha\{1 - (1 - e^{-\lambda x})^\beta\}]^2\}}; x \geq 0, (\alpha, \beta, \lambda) > 0 \tag{11}$$

Reversed Hazard function

The reversed hazard function of ATGE can be stated as

$$Rh(x) = \frac{f(x)}{F(x)} = \frac{\alpha\beta\lambda e^{-\lambda x} (1 - e^{-\lambda x})^{\beta-1}}{\{\arctan(\alpha) - \arctan[\alpha\{1 - (1 - e^{-\lambda x})^\beta\}]\}\{1 + [\alpha\{1 - (1 - e^{-\lambda x})^\beta\}]^2\}}; x \geq 0, (\alpha, \beta, \lambda) > 0 \tag{12}$$

In Fig 1, Illustration of the graphs of the PDF and failure rate function of ATGE model for varying α , and β values.

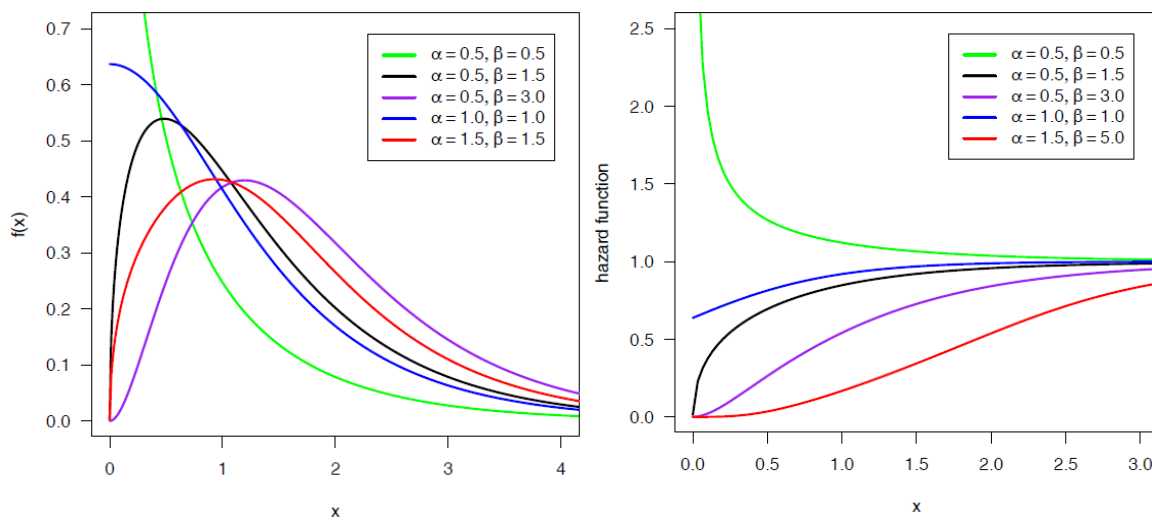


Fig 1. Graphs of hazard function (left) and PDF (right) for varying of α and β values.

Quantile function:

The quantile function of ATGE distribution where p follows uniform distribution (0,1) is

$$x_p = -\frac{1}{\lambda} \ln\left[1 - \left\{1 - \frac{1}{\alpha} \tan\left[(1-p) \arctan(\alpha)\right]\right\}^{1/\beta}\right]; 0 < p < 1 \tag{13}$$

Random deviation generation

$$x_u = -\frac{1}{\lambda} \ln\left[1 - \left\{1 - \frac{1}{\alpha} \tan\left[(1-u) \arctan(\alpha)\right]\right\}^{1/\beta}\right]; 0 < u < 1 \tag{14}$$

Skewness and Kurtosis:

Coefficient of skewness of Bowley based on octiles is

$$\Upsilon_{sk} = \frac{Q(0.75) + Q(0.25) - 2Q(0.5)}{Q(0.75) - Q(0.25)}, \text{ and} \tag{15}$$

Coefficient of kurtosis centered on octiles given by (Moors, 1988) is

$$M_{Ku} = \frac{Q(7/8) - Q(5/8) + Q(3/8) - Q(1/8)}{Q(3/4) - Q(1/4)} \tag{16}$$



III. METHODS OF ESTIMATION

3.1. Maximum Likelihood Estimation (MLE) Method

An estimator helps in approximation of the unknown parameters with the help of the measurements. In this paper, we have demonstrated the one of the well-known method called maximum likelihood method of estimation. Here, we have discussed the MLEs of the ATGE distribution. Suppose random sample $\underline{x} = (x_1, \dots, x_n)$ of size n be drawn from ATGE(α, β, λ), then the log likelihood function can be defined as,

$$l = n \ln \alpha + n \ln \beta + n \ln \lambda - n \ln \{ \arctan(\alpha) \} - \lambda \sum_{i=1}^n x_i - (\beta - 1) \sum_{i=1}^n \ln(1 - e^{-\lambda x_i}) - \sum_{i=1}^n \ln \{ 1 + [\alpha \{ 1 - (1 - e^{-\lambda x_i})^\beta \}]^2 \} \quad (17)$$

By differentiating (17) w.r.t. unknown parameters α, β and λ , we get

$$\begin{aligned} \frac{\partial l}{\partial \alpha} &= \frac{n}{\alpha} - \frac{n}{\arctan(\alpha)[1 + \alpha^2]} - 2\alpha \sum_{i=1}^n \frac{\{1 - (1 - e^{-\lambda x_i})^\beta\}^2}{1 + [\alpha \{1 - (1 - e^{-\lambda x_i})^\beta\}]^2} \\ \frac{\partial l}{\partial \beta} &= \frac{n}{\beta} + \sum_{i=1}^n \ln(1 - e^{-\lambda x_i}) + 2\alpha \sum_{i=1}^n \frac{\{1 - (1 - e^{-\lambda x_i})^\beta\} (1 - e^{-\lambda x_i})^\beta \ln(1 - e^{-\lambda x_i})}{1 + [\alpha \{1 - (1 - e^{-\lambda x_i})^\beta\}]^2} \\ \frac{\partial l}{\partial \lambda} &= \frac{n}{\lambda} - \sum_{i=1}^n x_i + (\beta - 1) \sum_{i=1}^n \frac{x_i e^{-\lambda x_i}}{(1 - e^{-\lambda x_i})} - 2\alpha \beta \sum_{i=1}^n \frac{x_i \{1 - (1 - e^{-\lambda x_i})^\beta\} (1 - e^{-\lambda x_i})^{\beta-1} e^{-\lambda x_i}}{1 + [\alpha \{1 - (1 - e^{-\lambda x_i})^\beta\}]^2} \end{aligned} \quad (18)$$

After equating these three equations to zero and solving for the unknown parameters (α, β, λ), we will obtain ML estimators of the ATGE distribution. Manually, it is tough to solve (18). Hence using appropriate computer software one can solve these equations. Representing the parameter vector by $\underline{\Theta} = (\alpha, \beta, \lambda)$ and the corresponding MLE of $\underline{\Theta}$ as $\underline{\hat{\Theta}} = (\hat{\alpha}, \hat{\beta}, \hat{\lambda})$, then the asymptotic normality outcomes in, $(\underline{\hat{\Theta}} - \underline{\Theta}) \rightarrow N_3 \left[0, (I(\underline{\Theta}))^{-1} \right]$, where $I(\underline{\Theta})$ denote the information matrix of Fisher given by

$$I(\underline{\Theta}) = - \begin{pmatrix} E \left(\frac{\partial^2 l}{\partial \alpha^2} \right) & E \left(\frac{\partial^2 l}{\partial \alpha \partial \beta} \right) & E \left(\frac{\partial^2 l}{\partial \alpha \partial \lambda} \right) \\ E \left(\frac{\partial^2 l}{\partial \beta \partial \alpha} \right) & E \left(\frac{\partial^2 l}{\partial \beta^2} \right) & E \left(\frac{\partial^2 l}{\partial \beta \partial \lambda} \right) \\ E \left(\frac{\partial^2 l}{\partial \alpha \partial \lambda} \right) & E \left(\frac{\partial^2 l}{\partial \lambda \partial \beta} \right) & E \left(\frac{\partial^2 l}{\partial \lambda^2} \right) \end{pmatrix}$$

In practice, we don't know $\underline{\Theta}$ hence it is unusable that the maximum likelihood estimator has an asymptotic variance $(I(\underline{\Theta}))^{-1}$. Thus, approximation of the asymptotic variance can be done by putting parameters' estimated value. The observed fisher information matrix $O(\underline{\Theta})$ is used as an information matrix $I(\underline{\Theta})$'s estimate given by

$$O(\underline{\Theta}) = - \begin{pmatrix} \frac{\partial^2 l}{\partial \alpha^2} & \frac{\partial^2 l}{\partial \alpha \partial \beta} & \frac{\partial^2 l}{\partial \alpha \partial \lambda} \\ \frac{\partial^2 l}{\partial \alpha \partial \beta} & \frac{\partial^2 l}{\partial \beta^2} & \frac{\partial^2 l}{\partial \beta \partial \lambda} \\ \frac{\partial^2 l}{\partial \alpha \partial \lambda} & \frac{\partial^2 l}{\partial \beta \partial \lambda} & \frac{\partial^2 l}{\partial \lambda^2} \end{pmatrix}_{(\hat{\alpha}, \hat{\beta}, \hat{\lambda})} = -H(\underline{\Theta})_{(\underline{\Theta} = \hat{\underline{\Theta}})}$$

where H denote the Hessian matrix.



The observed information matrix can be constructed with the help of the Newton-Raphson method maximizing the likelihood. Then, the var-covariance matrix is,

$$\left[-H(\Theta)_{(\Theta=\hat{\Theta})} \right]^{-1} = \begin{pmatrix} \text{var}(\hat{\alpha}) & \text{cov}(\hat{\alpha}, \hat{\beta}) & \text{cov}(\hat{\alpha}, \hat{\lambda}) \\ \text{cov}(\hat{\alpha}, \hat{\beta}) & \text{var}(\hat{\beta}) & \text{cov}(\hat{\lambda}, \hat{\beta}) \\ \text{cov}(\hat{\alpha}, \hat{\lambda}) & \text{cov}(\hat{\lambda}, \hat{\beta}) & \text{var}(\hat{\lambda}) \end{pmatrix} \tag{19}$$

Hence, an approximate 100(1- α) % confidence intervals for estimating α, β, λ can be built from the asymptotic normality of MLEs as $\hat{\alpha} \pm Z_{\alpha/2} \sqrt{\text{var}(\hat{\alpha})}$, $\hat{\beta} \pm Z_{\alpha/2} \sqrt{\text{var}(\hat{\beta})}$ and $\hat{\lambda} \pm Z_{\alpha/2} \sqrt{\text{var}(\hat{\lambda})}$, where upper percentile of standard normal variate is denoted by $Z_{\alpha/2}$

3.2. Method of Least-Square Estimation (LSE)

The another estimation method we have used is least-square estimation introduced by Swain et al. (1988) to estimate ATGE distribution's α, β and λ which is calculated with minimization of

$$M(X; \alpha, \beta, \lambda) = \sum_{i=1}^n \left[F(X_i) - \frac{i}{n+1} \right]^2 \tag{20}$$

w.r.t. α, β and λ .

From a distribution function $F(\cdot)$, suppose $F(X_i)$ represents ordered random variables $(X_{(1)} < X_{(2)} < \dots < X_{(n)})$'s CDF and random sample is denoted as $\{X_1, X_2, \dots, X_n\}$ with "n" size. The LSEs of the $(\hat{\alpha}, \hat{\beta}, \text{and } \hat{\lambda})$ is acquired with minimization of

$$M(X; \alpha, \beta, \lambda) = \sum_{i=1}^n \left[1 - \frac{\arctan[\alpha\{1 - (1 - e^{-\lambda x})^\beta\}]}{\arctan(\alpha)} - \frac{i}{n+1} \right]^2; x \geq 0, (\alpha, \beta, \lambda) > 0. \tag{21}$$

w.r.t. α, β and λ .

Differentiating (21) w.r.t. α, β and λ , we obtain

$$\begin{aligned} \frac{\partial M}{\partial \alpha} &= -2 \sum_{i=1}^n \left[1 - \frac{\arctan[\alpha w(x_i)]}{\arctan(\alpha)} - \frac{i}{n+1} \right] \left[\frac{w(x_i)}{(\alpha^2 \{w(x_i)\}^2 + 1) \arctan(\alpha)} - \frac{\arctan[\alpha w(x_i)]}{(\alpha^2 + 1) \arctan^2(\alpha)} \right] \\ \frac{\partial M}{\partial \beta} &= \frac{2\alpha}{\arctan(\alpha)} \sum_{i=1}^n \left[1 - \frac{\arctan[\alpha w(x_i)]}{\arctan(\alpha)} - \frac{i}{n+1} \right] \left[\frac{(1 - e^{-\lambda x_i})^\beta \ln(1 - e^{-\lambda x_i})}{(\alpha^2 \{w(x_i)\}^2 + 1)} \right] \\ \frac{\partial M}{\partial \lambda} &= \frac{2\alpha\beta}{\arctan(\alpha)} \sum_{i=1}^n \left[1 - \frac{\arctan[\alpha w(x_i)]}{\arctan(\alpha)} - \frac{i}{n+1} \right] \left[\frac{x_i (1 - e^{-\lambda x_i})^{\beta-1} e^{-\lambda x_i}}{(\alpha^2 \{w(x_i)\}^2 + 1)} \right] \end{aligned}$$

Where $w(x_i) = 1 - (1 - e^{-\lambda x_i})^\beta$

Likewise, the weighted least square estimators can be found with minimization of

$$M(X; \alpha, \beta, \lambda) = \sum_{i=1}^n w_i \left[F(X_{(i)}) - \frac{i}{n+1} \right]^2$$

w.r.t. α, β and λ , where the weight w_i is defined by $w_i = \frac{1}{\text{Var}(X_{(i)})} = \frac{(n+1)^2 (n+2)}{i(n-i+1)}$

Therefore, by minimizing function (22), the weighted least square estimators of α, β and λ can be found.



$$M(X; \alpha, \beta, \lambda) = \sum_{i=1}^n \frac{(n+1)^2 (n+2)}{i(n-i+1)} \left[1 - \frac{\arctan[\alpha\{1-(1-e^{-\lambda x_i})^\beta\}]}{\arctan(\alpha)} - \frac{i}{n+1} \right]^2 \tag{22}$$

w.r.t. α, β and λ .

3.3. Method of Cramer-Von-Mises estimation (CVME)

The Cramer-Von-Mises estimators of α, β and λ are found with minimization of the following function

$$D(X; \alpha, \beta, \lambda) = \frac{1}{12n} + \sum_{i=1}^n \left[F(x_{i:n} | \alpha, \beta, \lambda) - \frac{2i-1}{2n} \right]^2 = \frac{1}{12n} + \sum_{i=1}^n \left[1 - \frac{\arctan[\alpha w(x_i)]}{\arctan(\alpha)} - \frac{2i-1}{2n} \right]^2 \tag{23}$$

Differentiating (23) w.r.t. α, β and λ , we get,

$$\frac{\partial D}{\partial \alpha} = -2 \sum_{i=1}^n \left[1 - \frac{\arctan[\alpha w(x_i)]}{\arctan(\alpha)} - \frac{2i-1}{2n} \right] \left[\frac{w(x_i)}{(\alpha^2 \{w(x_i)\}^2 + 1) \arctan(\alpha)} - \frac{\arctan[\alpha w(x_i)]}{(\alpha^2 + 1) \arctan^2(\alpha)} \right]$$

$$\frac{\partial D}{\partial \beta} = \frac{2\alpha\beta}{\arctan(\alpha)} \sum_{i=1}^n \left[1 - \frac{\arctan[\alpha w(x_i)]}{\arctan(\alpha)} - \frac{2i-1}{2n} \right] \left[\frac{(1-e^{-\lambda x_i})^{\beta-1}}{(\alpha^2 \{w(x_i)\}^2 + 1)} \right]$$

$$\frac{\partial D}{\partial \lambda} = \frac{2\alpha\beta}{\arctan(\alpha)} \sum_{i=1}^n \left[1 - \frac{\arctan[\alpha w(x_i)]}{\arctan(\alpha)} - \frac{2i-1}{2n} \right] \left[\frac{x_i (1-e^{-\lambda x_i})^{\beta-1} e^{-\lambda x_i}}{(\alpha^2 \{w(x_i)\}^2 + 1)} \right]$$

Where $w(x_i) = 1 - (1 - e^{-\lambda x_i})^\beta$

By solving $\frac{\partial D}{\partial \alpha} = 0, \frac{\partial D}{\partial \beta} = 0$ and $\frac{\partial D}{\partial \lambda} = 0$ simultaneously, the CVM estimators can be obtained.

IV. APPLICATIONS TO REAL DATA

In this segment, we demonstrate the applicability of arctan generalized exponential distribution using two real sets of data applied by earlier researchers.

Dataset-I

The data below are from an accelerated life test of 59 conductors with failure times in hours (Nelson & Doganaksoy, 1995). The dataset has no censored observations.

9.289, 6.545, 6.956, 7.543, 5.459, 6.492, 4.706, 8.120, 2.997, 8.687, 6.129, 8.591, 5.381, 11.038, 4.288, 6.958, 4.137, 6.522, 7.495, 7.459, 6.538, 6.573, 6.087, 5.589, 5.807, 8.532, 6.725, 9.663, 7.024, 6.369, 8.336, 7.945, 9.218, 6.869, 4.700, 6.352, 6.948, 5.009, 9.254, 7.398, 7.489, 6.033, 7.496, 10.092, 7.974, 4.531, 7.683, 8.799, 7.224, 6.923, 7.365, 5.640, 7.937, 5.434, 6.515, 6.071, 6.476, 5.923, 10.491.

The MLEs are calculated by using the in platform of R via optim() function (R Core Team, 2020) and (Ming Hui, 2019) with maximization of likelihood function (17). We have obtained Log-Likelihood value is $l = -111.2103$ and the MLE's with their standard errors (SE) and 95% confidence interval for α, β and λ are presented in Table 1.

Table 1
MLE and SE and 95% confidence interval for α, β and λ

Parameter	MLE	SE	95% ACI
alpha	13.5030	6.6199	(0.5280, 26.478)
beta	33.6745	5.4436	(23.0050, 44.3440)
lambda	0.8916	0.0452	(0.8030, 0.9802)

The graphs of the profile log-likelihood function of α, β and λ are plotted in Figure 2 (Kumar & Ligges, 2011) and we observed that the MLEs are exclusive.

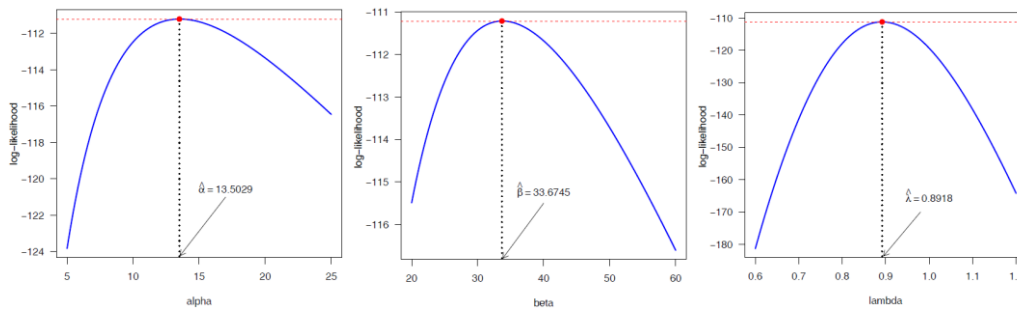


Figure 2. Graphs of profile log-likelihood function of α , β , and λ (dataset-I).

In Figure 3, we have displayed the Q-Q plot and P-P plot of ATGE distribution.

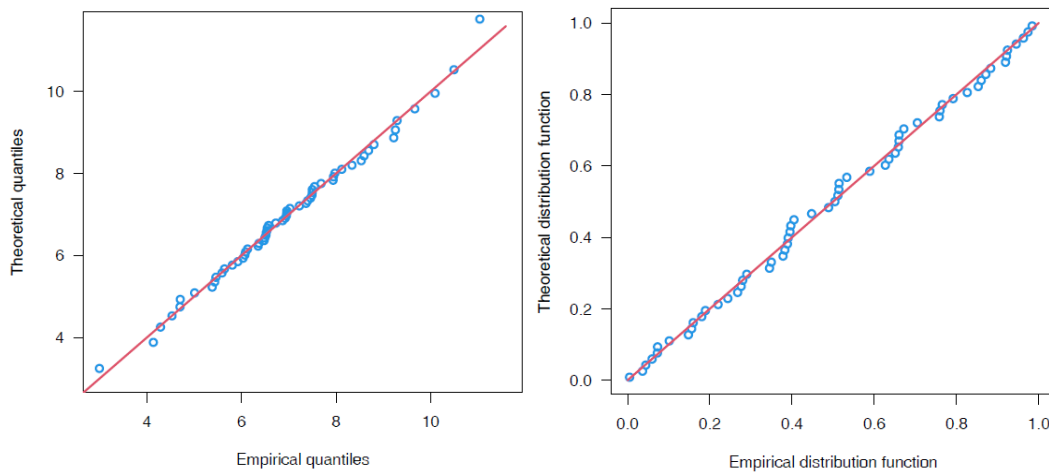


Figure 3. The Q-Q plot (left panel) and P-P plot (right panel) of ATGE distribution (dataset-I).

Dataset-II

The data (Birnbau & Saunders, 1969) given below represents the fatigue life of 6061-T6 aluminum coupons cut parallel to the direction of rolling and oscillated at 18 cycles per seconds (cps) which has 101 observations with maximum stress per cycle 31,000 psi.

116, 120, 120, 119, 120, 123, 124, 124, 124, 124, 124, 96, 97, 99, 100, 103, 104, 104, 114, 112, 113, 164, 166, 166114, 114, 105, 132, 132, 144, 144, 145, 146, 129, 130, 132, 107, 108, 108, 108, 109, 109, 112, 168, 136128, 128, 129, 70, 90, 137, 138, 157, 138, 134, 134, 134, 134, 138, 139, 139, 141, 141, 142, 142, 142, 142, 142, 174, 196, 212, 130, 130, 131, 131, 121, 121, 131, 131, 133, 134, 136, 148, 148, 149, 157, 157, 158, 159, 162, 151, 151, 152, 155, 156, 157, 163, 163, 170

We have achieved Log-Likelihood value is $l = -455.6293$ and the MLE’s with their standard errors (SE) and 95% confidence interval for α , β , and λ are presented in Table 2.

Table 2
MLE and SE and 95% confidence interval for α , β and λ

Parameter	MLE	SE	95% ACI
alpha	82.1638	2.4764	(77.3100, 87.0175)
beta	94.0816	3.0431	(88.1169, 100.0462)
lambda	0.0672	0.0011	(0.0650, 0.0693)



Log-likelihood function's plot for α , β and λ has been illustrated in Figure 4 (Kumar & Ligges, 2011) and found that the ML estimates can be calculated uniquely.

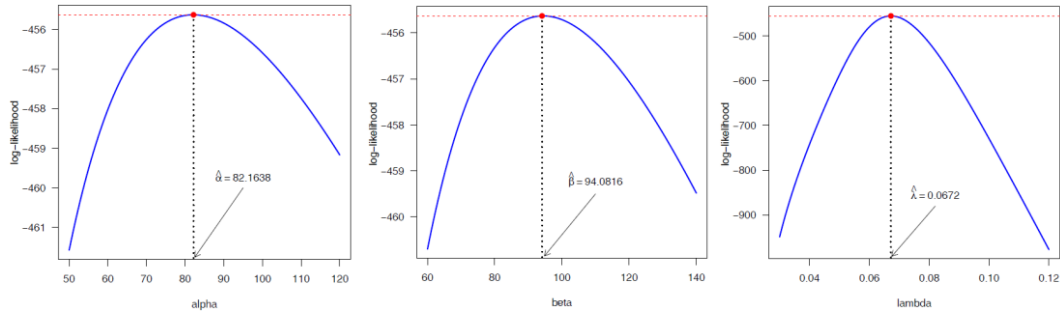


Figure 4. Graph of profile log-likelihood function of α , β , and λ (dataset-II).

In figure 5, we have displayed the Q-Q plot and P-P plot of ATGE distribution which shows good fit to the dataset

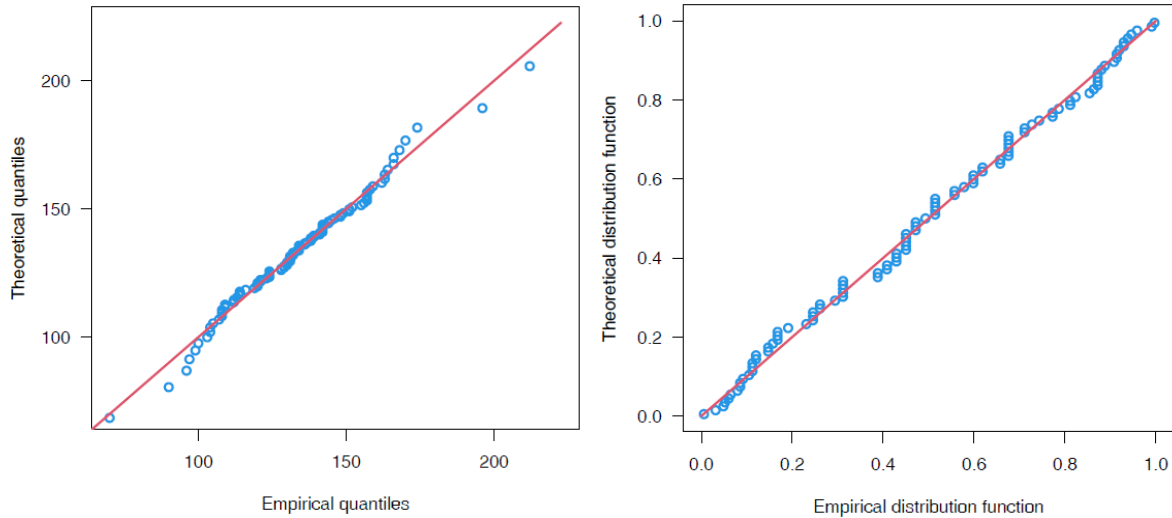


Figure 5. The Q-Q plot (left panel) and P-P plot (right panel) of ATGE distribution (dataset-II).

In Table 3 and Table 4, we have given the estimated values of the parameters of ATGE distribution using MLE, LSE and CVE methods and their corresponding negative log-likelihood, AIC and KS criterion for the data set I and II respectively.

Table 3
Estimated parameters, log-likelihood AIC and KS (data set-I)

Method of Estimation	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\lambda}$	-LL	AIC	KS(p-value)
MLE	13.5030	33.6745	0.8916	111.2103	228.4205	0.0529(0.9935)
LSE	13.7375	26.3363	0.8583	111.2835	228.5670	0.0498(0.9971)
CVE	15.0284	28.9459	0.8845	111.2266	228.4531	0.0523(0.9944)



Table 4
Estimated parameters, log-likelihood AIC and KS (data set-II)

Method of Estimation	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\lambda}$	-LL	AIC	KS(p-value)
MLE	82.1638	94.0816	0.0672	455.6293	917.2587	0.0501(0.9616)
LSE	127.6378	40.6934	0.0642	455.9277	917.8555	0.0466(0.9809)
CVE	83.4939	70.7746	0.0652	455.7335	917.4669	0.0460(0.9832)

In figure 6, we have plotted the histogram and the density function of fitted distributions and Q-Q plot of estimation methods MLE, LSE and CVM for data sets I and II.

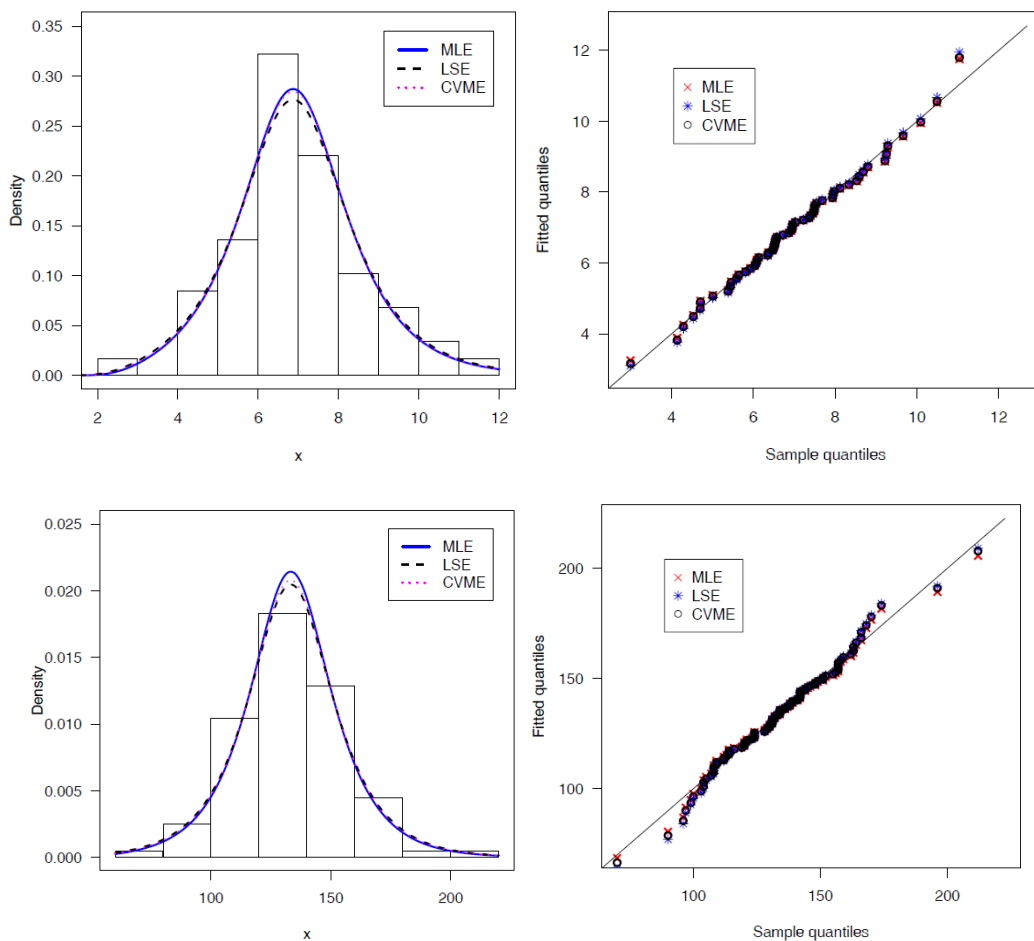


Figure 6. The Histogram and the density function of fitted distributions of estimation methods MLE, LSE and CVM (first column) and fitted quantiles and sample quantiles (second column) of ATGE distribution of datasets I and II respectively.

To demonstrate the goodness of fit of the ATGE distribution, following distributions for comparison purpose are selected which are listed below.

A. Gompertz distribution (GZ):

The probability density function of Gompertz distribution (Murthy et al., 2003) with parameters α and θ is



$$f_{GZ}(x) = \theta e^{\alpha x} \exp\left\{\frac{\theta}{\alpha}(1 - e^{\alpha x})\right\}; x \geq 0, \theta > 0, -\infty < \alpha < \infty.$$

B. Generalized Gompertz distribution

The PDF of Gompertz distribution (El-Gohary et al., 2013) with parameters α, λ and θ is

$$f_{GGZ}(x) = \theta \lambda e^{\alpha x} e^{-\frac{\lambda}{\alpha}(e^{\alpha x}-1)} \left[1 - \exp\left(-\frac{\lambda}{\alpha}(e^{\alpha x}-1)\right)\right]^{\theta-1}; \lambda, \theta > 0, \alpha \geq 0, x \geq 0$$

C. Generalized Exponential Extension (GEE) distribution:

The PDF of GEE distribution with parameters α, β and λ introduced by (Lemonte, 2013) having properties of upside down bathtub-shaped hazard function is

$$f_{GEE}(x; \alpha, \beta, \lambda) = \alpha \beta \lambda (1 + \lambda x)^{\alpha-1} \exp\{1 - (1 + \lambda x)^\alpha\} \left[1 - \exp\{1 - (1 + \lambda x)^\alpha\}\right]^{\beta-1}; x \geq 0.$$

D. Exponential power (EP) distribution:

The PDF of EP distribution introduced by (Smith & Bain, 1975) is

$$f_{EP}(x) = \alpha \lambda^\alpha x^{\alpha-1} e^{(\lambda x)^\alpha} \exp\{1 - e^{(\lambda x)^\alpha}\}; (\alpha, \lambda) > 0, x \geq 0.$$

where α and λ are the shape and scale parameters, respectively.

E. Generalized Exponential (GE) distribution:

The PDF of GE distribution introduced by (Gupta & Kundu, 1999) is.

$$f_{GE}(x; \alpha, \lambda) = \alpha \lambda e^{-\lambda x} \{1 - e^{-\lambda x}\}^{\alpha-1}; (\alpha, \lambda) > 0, x > 0$$

For the assessment of potentiality of the proposed novel distribution, we have determined the Akaike information criterion (AIC), Corrected Akaike information criterion (CAIC), Hannan-Quinn information criterion (HQIC) and Bayesian information criterion (BIC) which are presented in Table 5 and Table 6 for data sets I and II.

Table 5
Log-likelihood (LL), AIC, BIC, CAIC and HQIC (dataset-I)

Distribution	-LL	AIC	BIC	CAIC	HQIC
ATGE	111.2103	228.4205	234.6531	228.8569	230.8535
GGZ	111.2992	228.5983	234.8309	229.0347	231.0313
GEE	111.4165	228.8330	235.0656	229.2693	231.2659
GE	114.9473	233.8946	238.0497	234.1089	235.5166
GZ	116.5015	237.0029	241.1580	237.2098	238.6249
EP	117.1740	238.3480	242.5031	238.5623	239.9700

Table 6
Log-likelihood (LL), AIC, BIC, CAIC and HQIC (dataset-II)

Distribution	-LL	AIC	BIC	CAIC	HQIC
ATGE	455.6293	917.2587	925.1040	917.5061	920.4347
GGZ	456.2442	918.4885	926.3338	918.7359	921.6645
GEE	456.4468	918.8935	926.7389	919.1409	922.0696
GE	463.7324	931.4648	936.6951	931.5873	933.5822
GZ	472.8862	949.7724	955.0026	949.8948	951.8897
EP	476.7897	957.5794	962.8096	957.6994	959.6967

For the both datasets, we have presented the Histogram and the density function of fitted distributions and Empirical distribution function with the estimated distribution function of ATGE and some selected well-known distributions are presented in Figure 7.

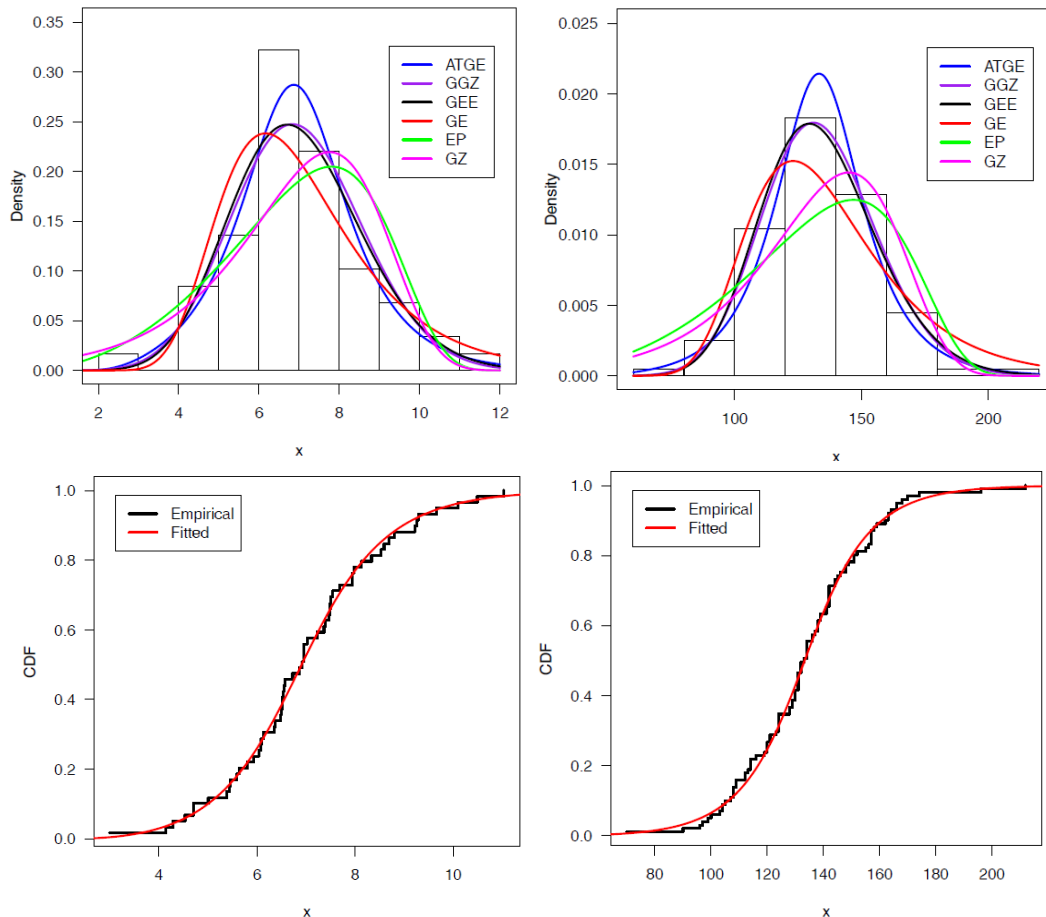


Figure 7. The Histogram and the density function of fitted distributions for data-I and data-II (first row) and Empirical distribution function with estimated distribution function for data-I and data-II (second row)

To compare the goodness-of-fit of the ATGE distribution with other well-known existing distributions, we have calculated the values of Kolmogorov-Simnrov (KS), the Anderson-Darling (W) and the Cramer-Von Mises (A^2) statistics in Table 7 and Table 8. It is observed that the ATGE distribution has the minimum value of the test statistic and higher p -value for the both datasets I and II, thus we accomplish that the ATGE distribution gets quite better fit and more consistent and reliable outcomes from others taken for comparison.

Table 7
The goodness-of-fit statistics and their corresponding p-value (dataset-I)

Distribution	KS(p-value)	W(p-value)	A^2 (p-value)
ATGE	0.0529(0.9935)	0.0205(0.9968)	0.1338(0.9995)
GGZ	0.0661(0.9437)	0.0354(0.9564)	0.2030(0.9896)
GEE	0.0633(0.9601)	0.0356(0.9557)	0.2052(0.9889)
GE	0.1042(0.5103)	0.1173(0.5079)	0.7368(0.5282)
GZ	0.1365(0.2021)	0.2398(0.2021)	1.3735(0.2098)
EP	0.1306(0.2464)	0.216 (0.2387)	1.3143(0.2277)

Table 8
The goodness-of-fit statistics and their corresponding p-value (dataset-II)

Distribution	KS(p-value)	W(p-value)	A ² (p-value)
ATGE	0.0501(0.9616)	0.0360(0.9535)	0.2748(0.9557)
GGZ	0.0653(0.7824)	0.0540(0.8526)	0.3445(0.9012)
GEE	0.0755(0.6132)	0.0682(0.7640)	0.4132(0.8352)
GE	0.1066(0.2014)	0.3112(0.1257)	2.0724(0.0840)
GZ	0.1248(0.0859)	0.4461(0.0545)	3.0362(0.0263)
EP	0.1378(0.0433)	0.6942(0.0130)	4.5057(0.0050)

V. CONCLUSION

In this work, we have generated and studied a novel three-parameter continuous model called arctan generalized exponential distribution. A comprehensive study of some distributional characteristics of the new distribution is presented build a clearer picture for the distribution purposed and we have found that the proposed novel model has flexible hazard rate function. Parameter estimation is carried out with MLE along with CVME and LSE. and it is found that MLEs are relatively good than LSEs and CVMs. Two real data sets are used to illustrate the flexibility, suitability and potentiality of the novel proposed distribution. To assess the validity of the proposed distribution, AICC, AIC, HQIC and BIC are also calculated and found that the proposed model is fairly better than other lifespan model taken into thought. We hope this novel proposed model may be an alternative in the field of reliability analysis, applied statistics and probability theory.

REFERENCES

- Birnbaum, Z.W., & Saunders, S.C. (1969). Estimation for a family of life distributions with applications to fatigue, *Journal of Applied Probability*, 6, 328 -347.
- Chaudhary, A. K. & Kumar, V. (2020). Half logistic exponential extension distribution with Properties and Applications. *International Journal of Recent Technology and Engineering (IJRTE)*, 8(3), 506-512.
- Chaudhary, A. K. & Kumar, V. (2020). A Bayesian analysis three-parameter complementary exponential power model via Markov Chain Monte Carlo simulation. *International Journal of Engineering and Advanced Technology (IJEAT)*, 10(2), 164-174.
- Chaudhary, A.K. & Kumar, V.(2020).A Study on Properties and Applications of Logistic Modified Exponential Distribution. *International Journal of Latest Trends In Engineering and Technology (IJLTET)*,18(1),19-29
- Chaudhary, A. K., Sapkota, L. P. & Kumar, V. (2020). Truncated Cauchy power-exponential distribution: Theory and Applications. *IOSR Journal of Mathematics (IOSR-JM)*, 16(6), 44-52.
- Chaudhary, A. K., Sapkota, L. P. & Kumar, V. (2020). Truncated Cauchy Power-Inverse Exponential distribution: Theory and Applications. *IOSR Journal of Mathematics (IOSR-JM)*, 16(4),Ser.IV, 12-23.
- El-Gohary, A., Alshamrani, A., & Al-Otaibi, A. N. (2013). The generalized Gompertz distribution. *Applied Mathematical Modelling*, 37(1-2), 13-24.
- Gómez-Déniz, E., & Calderín-Ojeda, E. (2015). Modelling insurance data with the Pareto ArcTan distribution. *ASTIN Bulletin: The Journal of the IAA*, 45(3), 639-660.
- Gupta, R. D., & Kundu, D. (1999). Theory & methods: Generalized exponential distributions. *Australian & New Zealand Journal of Statistics*, 41(2), 173-188.
- Joshi, R. K (2015). An Extension of Exponential Distribution: Theory and Applications, *Journal of National Academy of Mathematics India*, 29, 99-108.
- Joshi, R. K. & Kumar, V. (2020). New Exponential Extension Poisson distribution: Properties and application. *International Journal of Mathematics and Statistics Invention (IJMSI)*, 8(9), 35-45.
- Joshi, R. K. & Kumar, V. (2020). Lindley exponential power distribution with Properties and Applications. *International Journal for Research in Applied Science & Engineering Technology (IJRASET)*, 8(10), 22-30.
- Joshi, R. K., Sapkota, L.P. & Kumar, V. (2020).The Logistic-Exponential Power Distribution with Statistical Properties and Applications, *International Journal of Emerging Technologies and Innovative Research*, 7(12), 629-641.



14. Kumar, V. and Ligges, U. (2011). reliaR: A package for some probability distributions, <http://cran.r-project.org/web/packages/reliaR/index.html>.
15. Lan, Y., & Leemis, L. M. (2008). The logistic–exponential survival distribution. *Naval Research Logistics (NRL)*, 55(3), 252-264.
16. Lemonte, A. J. (2013). A new exponential-type distribution with constant, decreasing, increasing, upside-down bathtub and bathtub-shaped failure rate function. *Computational Statistics & Data Analysis*, 62, 149-170.
17. Marshall, A. W. and Olkin, I. (2007). *Life Distributions: Structure of Nonparametric, Semiparametric and Parametric Families*, Springer, New York.
18. Ming Hui, E. G. (2019). *Learn R for applied statistics*. Springer, New York.
19. Moors, J. J. A. (1988). A quantile alternative for kurtosis. *Journal of the Royal Statistical Society: Series D (The Statistician)*, 37(1), 25-32.
20. Murthy, D.N.P., Xie, M. and Jiang, R. (2003). *Weibull Models*, Wiley, New York
21. Nadarajah, S., & Haghghi, F. (2011). An extension of the exponential distribution. *Statistics*, 45(6), 543-558.
22. Nelson, W., & Doganaksoy, N. (1995). Statistical analysis of life or strength data from specimens of various sizes using the power-(log) normal model. *Recent Advances in Life-Testing and Reliability*, 377-408.
23. Pham, H., & Lai, C. D. (2007). On recent generalizations of the Weibull distribution. *IEEE transactions on reliability*, 56(3), 454-458.
24. R Core Team (2020). R: A language and environment for statistical computing. R Foundation for Statistical Computing, Vienna, Austria. URL <https://www.R-project.org/>.
25. Smith, R. M., & Bain, L. J. (1975). An exponential power life-testing distribution. *Communications in Statistics-Theory and Methods*, 4(5), 469-481.
26. Swain, J. J., Venkatraman, S. & Wilson, J. R. (1988). Least-squares estimation of distribution functions in Johnson's translation system. *Journal of Statistical Computation and Simulation*, 29(4), 271–297.



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