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Two New Generalized Measures of Intuitionistic Fuzzy Directed Divergence

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ABSTRACT: In the existing literature of information theory, there is a huge availability of measures of fuzzy divergence each with its own merits and limitations. Keeping in view the flexibility in the system and application areas and by using the concept of intuitionistic fuzzy set we have introduced two new generalized parametric measures of intuitionistic fuzzy directed divergence and studied there important and interesting properties.

KEY WORDS: measure of entropy, directed divergence, intuitionistic fuzzy entropy, intuitionistic fuzzy directed divergence, information theory.

I. INTRODUCTION

A measure $D(P:Q)$ of divergence or cross entropy or directed divergence is found to be very important in Mathematical, Physical and Biological sciences. This measure is probabilistic in nature and is defined as the discrepancy in the probability distribution P from another probability distribution Q . In some sense it measures the distance of P from Q . The most important and useful measure of directed divergence is obtained by Kullback and Leibler of probability distribution $P = (p_1, p_2, \dots, p_n)$ from the probability distribution $Q = (q_1, q_2, \dots, q_n)$ as

$$D(P:Q) = \sum_{i=1}^n p_i \log \frac{p_i}{q_i} \quad (1)$$

Let A and B be two standard fuzzy sets with same supporting points x_1, x_2, \dots, x_n and with fuzzy vectors $\mu_A(x_1), \mu_A(x_2), \dots, \mu_A(x_n)$ and $\mu_B(x_1), \mu_B(x_2), \dots, \mu_B(x_n)$. The simplest measure of fuzzy directed divergence as suggested by Bhandari and Pal (1993), is

$$D(A:B) = \sum_{i=1}^n \mu_A(x_i) \log \frac{\mu_A(x_i)}{\mu_B(x_i)} + \sum_{i=1}^n (1 - \mu_A(x_i)) \log \frac{(1 - \mu_A(x_i))}{(1 - \mu_B(x_i))} \quad (2)$$

satisfying the conditions:

1. $D(A:B) \geq 0$
2. $D(A:B) = 0$ iff $A = B$
3. $D(A:B) = D(B:A)$
4. $D(A:B)$ is a convex function of $\mu_A(x_i)$

later kapur [5],[6] introduced a number of trigonometric hyperbolic and exponential measures of fuzzy entropy and fuzzy directed divergence. In section 2 and 3 we introduce some new trigonometric, hyperbolic and exponential measures of intuitionistic fuzzy entropy and measures of intuitionistic fuzzy directed divergence. Now we define the concept of intuitionistic fuzzy set and then discuss the properties of intuitionistic fuzzy entropy and intuitionistic fuzzy directed divergence.

The concept of intuitionistic fuzzy set was first time given by K. T. Atanasav (1983) as



INTUITIONISTIC FUZZY SET

Let a set E be fixed. An intuitionistic fuzzy set (IFs) A of E is an object having the form $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in E \}$ where the function $\mu_A: E \rightarrow [0,1]$ and $\nu_A: E \rightarrow [0,1]$ define respectively the degree of membership and degree of non membership of the element $x \in E$ to the set A, which is a subset of E and for every $x \in E$, $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

Conditions for measures of intuitionistic fuzzy directed divergence:- Let A and B be two standard intuitionistic fuzzy sets with same supporting points x_1, x_2, \dots, x_n with membership $\mu_A(x_1), \mu_A(x_2), \dots, \mu_A(x_n)$ and $\mu_B(x_1), \mu_B(x_2), \dots, \mu_B(x_n)$ and non membership $\nu_A(x_1), \nu_A(x_2), \dots, \nu_A(x_n)$ and $\nu_B(x_1), \nu_B(x_2), \dots, \nu_B(x_n)$. The measure of intuitionistic fuzzy directed divergence is denoted by $D(A: B)$ satisfying the following conditions.

1. $D(A: B) \geq 0$
2. $D(A: B) = 0$ iff $A = B$
3. $D(A: B)$ is a convex function of $\mu_A(x_i)$ and $\mu_B(x_i)$.

In the next section, we introduce some new measures of intuitionistic fuzzy directed divergence depending upon some real parameters.

II. NEW GENERALISED MEASURE OF INTUITIONISTIC FUZZY DIVERGENCE

We propose the following measures of intuitionistic fuzzy directed divergence:

$$D_1(A: B) = -\frac{1}{\alpha} \sum_{i=1}^n \left[2 - \left(\frac{\mu_A(x_i)}{\mu_B(x_i)} \right)^{\alpha \mu_A(x_i)} - \left(\frac{\nu_A(x_i)}{\nu_B(x_i)} \right)^{\alpha \nu_A(x_i)} \right], \alpha \neq 0 \tag{2.1}$$

To prove that (2.1) is a correct measure of divergence, we study the following properties:

1. $D_1(A: B) \geq 0$
2. $D_1(A: B) = 0$ iff $A = B$
3. $D_1(A: B)$ is a convex function of $\mu_A(x_i)$ and $\mu_B(x_i)$.

$$\frac{\partial^2 D_1(A: B)}{\partial \mu_A(x_i)^2} = \sum_{i=1}^n \left[\left(\frac{\mu_A(x_i)}{\mu_B(x_i)} \right)^{\alpha \mu_A(x_i)} \alpha \left(1 + \log \frac{\mu_A(x_i)}{\mu_B(x_i)} \right)^2 + \left(\frac{\mu_A(x_i)}{\mu_B(x_i)} \right)^{\alpha \mu_A(x_i)} \frac{1}{\mu_A(x_i)} \right] \geq 0$$

Which shows that $D_1(A: B)$ is a convex function of $\mu_A(x_i)$.

Similarly, we have

$$\frac{\partial^2 D_1(A: B)}{\partial \mu_B(x_i)^2} = \sum_{i=1}^n \frac{\mu_A(x_i)^{(1+\alpha \mu_A(x_i))}}{\mu_B(x_i)^{(2+\alpha \mu_A(x_i))}} (\alpha \mu_A(x_i) + 1) \geq 0$$

Which shows that $D_1(A: B)$ is a convex function of $\mu_B(x_i)$.

Under the above properties the mathematical expression introduced in (2.1) represents a valid measure of intuitionistic fuzzy directed divergence.

Next we propose the following measures of intuitionistic fuzzy directed divergence:

$$D_2(A: B) = -\frac{1}{\alpha} \sum_{i=1}^n \left[4 - \left(\frac{\mu_A(x_i)}{\mu_B(x_i)} \right)^{\alpha \mu_A(x_i)} - \left(\frac{\nu_A(x_i)}{\nu_B(x_i)} \right)^{\alpha \nu_A(x_i)} - \frac{\mu_A(x_i)}{\mu_B(x_i)} + \log \frac{\mu_A(x_i)}{\mu_B(x_i)} - \frac{\nu_A(x_i)}{\nu_B(x_i)} + \log \frac{\nu_A(x_i)}{\nu_B(x_i)} \right] \tag{2.2}$$

To prove that $D_1(A: B)$ is a correct measure of directed divergence we prove that following properties:

1. $D_2(A: B) \geq 0$

$$\frac{\partial^2 D_2(A: B)}{\partial \mu_A(x_i)^2} = -\frac{1}{\alpha} \sum_{i=1}^n \left[-\left(\frac{\mu_A(x_i)}{\mu_B(x_i)}\right)^{\alpha \mu_A(x_i)} \frac{\alpha}{\mu_A(x_i)} - \left(\frac{\mu_A(x_i)}{\mu_B(x_i)}\right)^{\alpha \mu_A(x_i)} \alpha^2 \left(1 + \log \frac{\mu_A(x_i)}{\mu_B(x_i)}\right)^2 - \frac{1}{(\mu_A(x_i))^2} \right] \geq 0$$

Which shows that $D_2(A: B)$ is a convex function of $\mu_A(x_i)$.

Similarly we can show that $D_2(A: B)$ is a convex function of $\mu_B(x_i)$

2. $D_2(A: B) = 0$ iff $A = B$

Under the above properties the mathematical expression introduced in (2.2) represents a valid measure of intuitionistic fuzzy directed divergence.

III. CONCLUSION

Two new measures of intuitionistic fuzzy directed divergence have been introduced and their validity has been checked.

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