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# Airplane Parts Modelling by Quadratic Hybrid ATE Bézier Curve

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**ABSTRACT:** In this paper a new kind of Quadratic Hybrid ATE Bézier Curve with a shape parameter  $\lambda$  is presented and the corresponding Hybrid ATE Bézier Bézier surfaces are defined. These curves not only inherit most properties of the usual quartic Bézier curves with the Bernstein basis in the polynomial space, but also enjoy some other advantageous properties for shape modelling. The shape parameter provides freedom in terms of design and shape control of the curve. Thus, we can construct smooth curves of almost any shape. The shape of the curve can be adjusted by altering the values of shape parameter while the control polygon is kept unchanged. These curves can be used as an efficient model for geometric design in the fields of CAGD. Quadratic Hybrid ATE Bézier Curve can be used for designing various parts of airplanes. A variety of Wing tips and tail fins shapes are constructed in this paper by using Quadratic Hybrid ATE Bézier Curve with a shape parameter  $\lambda$ .

**KEYWORDS:** Hybrid ATE Bézier basis function, Hybrid ATE Bézier curve, Shape parameter, Wingtips, Tail fins.

## I. INTRODUCTION

One of the most noticeable features of aircraft is the variety in their wingtip and tail fins shapes, see [1]. Wingtips and tail fins can modify efficiency and handling depending on their shapes. Bézier curves and surfaces are the basic tools for modelling in Computer Aided Geometric Designing (CAGD) and Computer Graphics (CG). Bézier polynomial has several applications in the fields of engineering, science and technology such as airplane designing, highway or railway rout designing, networks, Computer aided design system, animation, environment design, robotics, communications and many other disciplines because it is easy to compute and is also very stable. Uses of Bézier curves continues to grow at a rapid rate. The classical Bézier curves have some limitations that their shape and position are fixed relative to their control polygon. Thus people attempt to find a solution of the problem in the non-polynomial function space. During the last few years, a major research focus has been the use of trigonometric functions or the blending of polynomial and trigonometric functions.

Trigonometric B-splines were first presented in [2] In recent years, several bases in new spaces other than the polynomial space have been proposed for geometric modelling in CAGD ; see [3], [4] and [5]. In [6] cubic trigonometric Bézier curve with two shape parameters were presented. In [7] a basis is constructed for  $C_m = span\{1, \cos t, \dots, \cos mt\}$ . In [8] a basis for the space of trigonometric polynomials  $\{1, \sin t, \cos t, \dots, \sin mt, \cos mt\}$  is constructed. Some bases are constructed in [9] for the spaces  $\{1, t, \cos t, \sin t, \cos 2t, \sin 2t\}$ ,  $\{1, t, t^2, \cos t, \sin t\}$  and  $\{1, t, \cos t, \sin t, t \cos t, t \sin t\}$ . In [10] C-Bézier curves are constructed in the space spanned by  $\{1, t, t^2, \dots, t^{n-2}, \sin t, \cos t\}$ . Non-uniform algebraic trigonometric B-splines (NUAT B-splines), are generated in [11] over the space spanned by  $\{1, t, t^2, \dots, t^{k-3}, \cos t, \sin t\}$  in which k is an arbitrary integer larger than or equal to 3. But all these curves do not have any shape parameter. In [12, 13] C-B-splines are presented in the space spanned by  $\{1, t, \sin t, \cos t\}$  with parameter  $\alpha$ . In [14], quadratic non-uniform algebraic trigonometric (NUAT) B-splines with multiple shape parameters is studied. The Bézier Curves with Two Shape Parameters is constructed in [15], In [16], the cubic trigonometric polynomial curve similar to the cubic Bézier curves is constructed. In [17], the Quadratic Trigonometric Polynomial Bézier Curves with a Shape Parameter are analysed. In [18], shape analysis of cubic trigonometric bézier curves with a shape parameter are studied. An extension of the Bézier model is studied in [19]. In [20] AE B Spline Curves and Surfaces for Geometric Modelling are discussed. In [21] and [22] quartic trigonometric Bézier curves with shape parameter is presented and the effect of shape parameter is studied. Some engineering symmetric revolutionary curves and symmetric rotation surfaces by using the GHT-Bézier curve is presented in [23].

In this paper a new kind of Quadratic Hybrid ATE Bézier Curve with a shape parameter is presented. The paper is organized as follows. In section II, Quadratic Hybrid ATE Bézier basis functions with a shape parameter are established and the properties of the basis functions are shown. In section III, Quadratic Hybrid ATE Bézier curves are given and some properties are discussed. By using shape parameter, shape control of the Quadratic Hybrid ATE Bézier curves is studied. In section IV, Example of Modelling of Wing tips and Tail fins of Airplane are presented. In section V, the representation of Quadratic Hybrid ATE Bézier surface has been shown. The conclusion is presented in Section VI.

## II. QUADRATIC HYBRID ATE BÉZIER BASIS FUNCTIONS

Firstly, the definition of Quadratic Hybrid ATE Bézier basis functions is given as follows.

### The construction of the Quadratic Hybrid ATE Bézier basis functions

**Definition 1.** For an arbitrarily selected real values of  $\lambda$  where  $\lambda \in [-2.5, 1]$ , the following three functions of  $t$  ( $t \in [0, 1]$ ) are defined as Hybrid ATE (Algebraic Trigonometric Exponential) Bézier basis functions with a shape parameter  $\lambda$ :

$$\begin{cases} b_0(t) = (1 - \sin \frac{\pi}{2} t) (1 - \lambda t) e^{-t} \\ b_1(t) = 1 - (1 - \sin \frac{\pi}{2} t) (1 - \lambda t) e^{-t} - (1 - \cos \frac{\pi}{2} t) (1 - \lambda + \lambda t) e^{(t-1)} \\ b_2(t) = (1 - \cos \frac{\pi}{2} t) (1 - \lambda + \lambda t) e^{(t-1)} \end{cases} \quad (1)$$

### The properties of the Quadratic Hybrid ATE Bézier basis functions

**Theorem 1.** The basis functions (1) have the following properties:

- (a) **Non-negativity:**  $b_i(t) \geq 0$  for  $i = 0, 1, 2$ .
- (b) **Partition of unity:**  $\sum_{i=0}^2 b_i(t) = 1$
- (c) **Symmetry:**  $b_i(t; \lambda) = b_{2-i}(1-t; \lambda)$ , for  $i = 0, 1, 2$ .
- (d) **Monotonicity:** For a given parameter  $t$ , as the shape parameter  $\lambda$  increases (or decreases),  $b_0(t)$ , and  $b_2(t)$  decreases (or increases) and  $b_1(t)$  increases (or decreases).

**Proof:** (a) For  $t \in [0, 1]$  and  $\lambda \in [-2.5, 1]$  then

$$0 \leq (1 - \sin \frac{\pi}{2} t) \leq 1, 0 \leq (1 - \cos \frac{\pi}{2} t) \leq 1,$$

$$0 \leq (1 - \lambda t) \leq 1, 0 \leq (1 - \lambda + \lambda t) \leq 1,$$

$$0 \leq e^{-t} \leq 1, 0 \leq e^{(t-1)} \leq 1$$

It is obvious that  $b_i(t) \geq 0$  for  $i = 0, 1, 2$ .

$$(b) \sum_{i=0}^2 b_i(t) = 1$$

The remaining cases follow obviously.

The curves of the Quadratic Hybrid ATE Bézier basis functions are shown in Figure 1 for  $\lambda = -2.5, -2.0, -1.5, -1.0, -0.5, 0, 0.5, 1$ . In this figure, the basis functions  $b_0(t)$ ,  $b_1(t)$  and  $b_2(t)$  are represented by red lines, green lines and blue lines respectively.



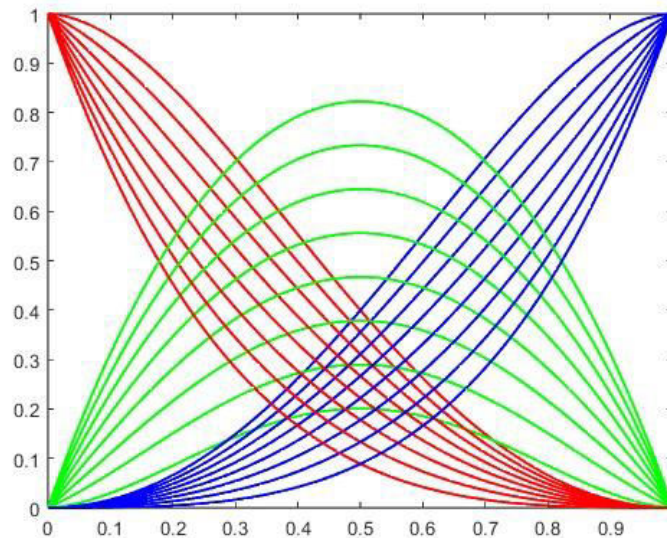


Fig. 1. Quadratic Hybrid ATE Bézier basis functions for different values of  $\lambda$

### III. QUADRATIC HYBRID ATE BÉZIER CURVE

#### The construction of the Quadratic Hybrid ATE Bézier curve

**Definition 2.** Given control points  $P_i = 0, 1, 2$  in  $R^2$  or  $R^3$ , then

$$r(t) = \sum_{i=0}^2 b_i(t)P_i, \quad t \in [0, 1], \lambda \in [-2.5, 1] \quad (2)$$

is called a Quadratic Hybrid ATE Bézier curve with a shape parameter  $\lambda$ .

From the definition of the basis function, some properties of the Quadratic Hybrid ATE Bézier curve can be obtained as follows:

**Theorem 2** The Quadratic Hybrid ATE Bézier curves (2) have the following properties:

a) *Terminal Properties:*  $r(0) = P_0, \quad r(1) = P_2,$  (3)

$$r'(0) = (P_1 - P_0) \left( \lambda + \frac{\pi}{2} + 1 \right), \quad r'(1) = (P_2 - P_1) \left( \lambda + \frac{\pi}{2} + 1 \right) \quad (4)$$

b) *Symmetry:*  $P_0, P_1, P_2$  and  $P_2, P_1, P_0$  define the same Quadratic Hybrid ATE Bézier curve in different parametrizations, i.e.,

$$r(t; \lambda; P_0, P_1, P_2) = r(1-t; \lambda; P_2, P_1, P_0); \quad t \in [0, 1], \lambda \in [-2.5, 1] \quad (5)$$

c) *Geometric invariance:* The shape of a Quadratic Hybrid ATE Bézier curve is independent of the choice of coordinates, i.e. (2) satisfies the following two equations:

$$\begin{aligned} r(t; \lambda; P_0 + q, P_1 + q, P_2 + q) &= r(t; \lambda; P_0, P_1, P_2) + q \\ r(t; \lambda; P_0 * T, P_1 * T, P_2 * T) &= r(t; \lambda; P_0, P_1, P_2) * T \end{aligned} \quad (7)$$

$$t \in [0, 1], \lambda \in [-2.5, 1]$$

where  $q$  is arbitrary vector in  $R^2$  or  $R^3$ , and  $T$  is an arbitrary  $d \times d$  matrix,  $d=2$  or  $3$ .

d) *Convex hull property:* The entire Quadratic Hybrid ATE Bézier curve segment lies inside its control polygon spanned by  $P_0, P_1, P_2$ .

### Shape Control of the Quadratic Hybrid ATE Bézier Curve

For  $t \in [0, 1]$ , we rewrite (2) as follows:

$$r(t) = \sum_{i=0}^2 P_i c_i(t) + \lambda \left(1 - \sin \frac{\pi}{2} t\right) t e^{-t} (P_1 - P_0) - \lambda \left(1 - \cos \frac{\pi}{2} t\right) (1 - t) e^{(t-1)} (P_2 - P_1) \tag{8}$$

where  $c_0(t) = \left(1 - \sin \frac{\pi}{2} t\right) e^{-t}$ ,  $c_1(t) = \left[1 - \left(1 - \sin \frac{\pi}{2} t\right) e^{-t} - \left(1 - \cos \frac{\pi}{2} t\right) e^{(t-1)}\right]$ ,  $c_2(t) = \left(1 - \cos \frac{\pi}{2} t\right) e^{(t-1)}$ .

Obviously, shape parameter  $\lambda$  affects the curve on the control edges  $(P_1 - P_0)$  and  $(P_2 - P_1)$ . Figure 3 shows a computed example of Quadratic Hybrid ATE Bézier Curves with different values of shape parameter  $\lambda$ . These curves are generated by setting  $P_0 = (0, 0)$ ,  $P_1 = (4, 4)$ ,  $P_2 = (8, 0)$  and  $\lambda = \lambda = -2.5, -2.0, -1.5, -1.0, -0.5, 0, 0.5, 1$ .

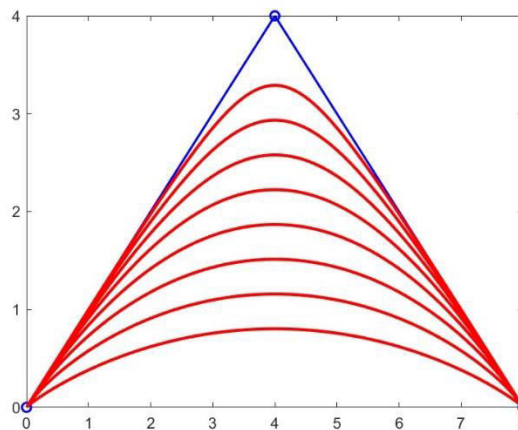


Figure 3: Example of Quadratic Hybrid ATE Bézier Curves with different values of shape parameter  $\lambda$

### IV. EXAMPLE: MODELLING OF WING TIPS AND TAIL FINS OF AIRPLANE

Any airplane that wants to fly needs its weight, together with the weight of the fuel, the occupants, and the cargo, lifted. Most of the lift needed to keep the aircraft in the air is produced by the wings. As the airplane moves through the air, there is a aerodynamic force present. The air resists the motion of the aircraft; this resistance force is called the drag of the airplane. In Figure 4, various types of rounded and sharp wing tips of Airplane are constructed to reduce drag by Quadratic Hybrid ATE Bézier Curves with different values of shape parameter  $\lambda$ . The corresponding control polygon is also shown in this figure, In Figure 5, various types of Tail fins of Airplane (Specially Jet Fighter) with their corresponding control polygon are constructed by Quadratic Hybrid ATE Bézier Curves with different values of shape parameter  $\lambda = -2, -1, 0, 0.5, 1$ .

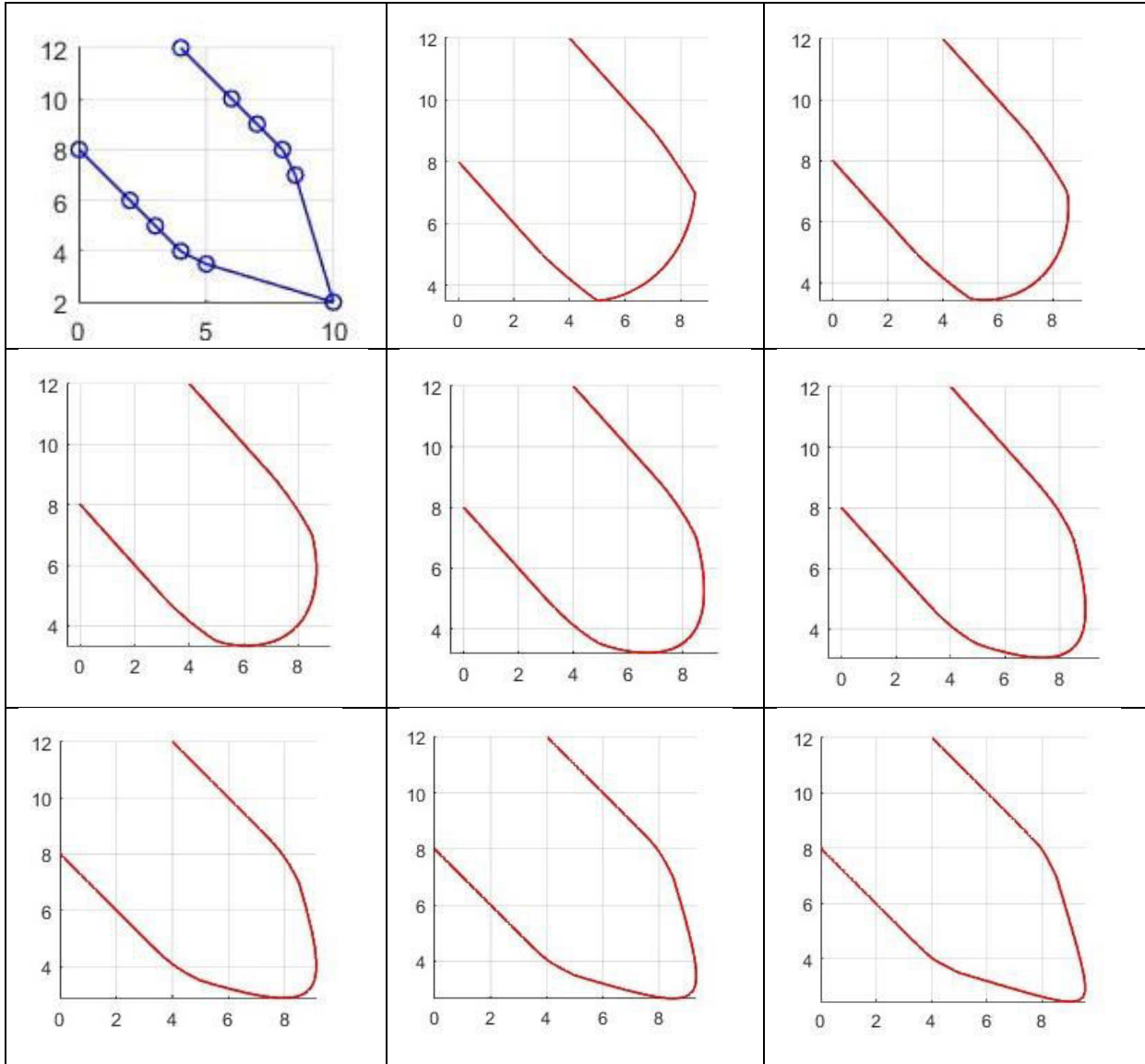


Figure 4: Various types of rounded and sharp wing tips of Airplane, constructed by Quadratic Hybrid ATE Bézier Curves

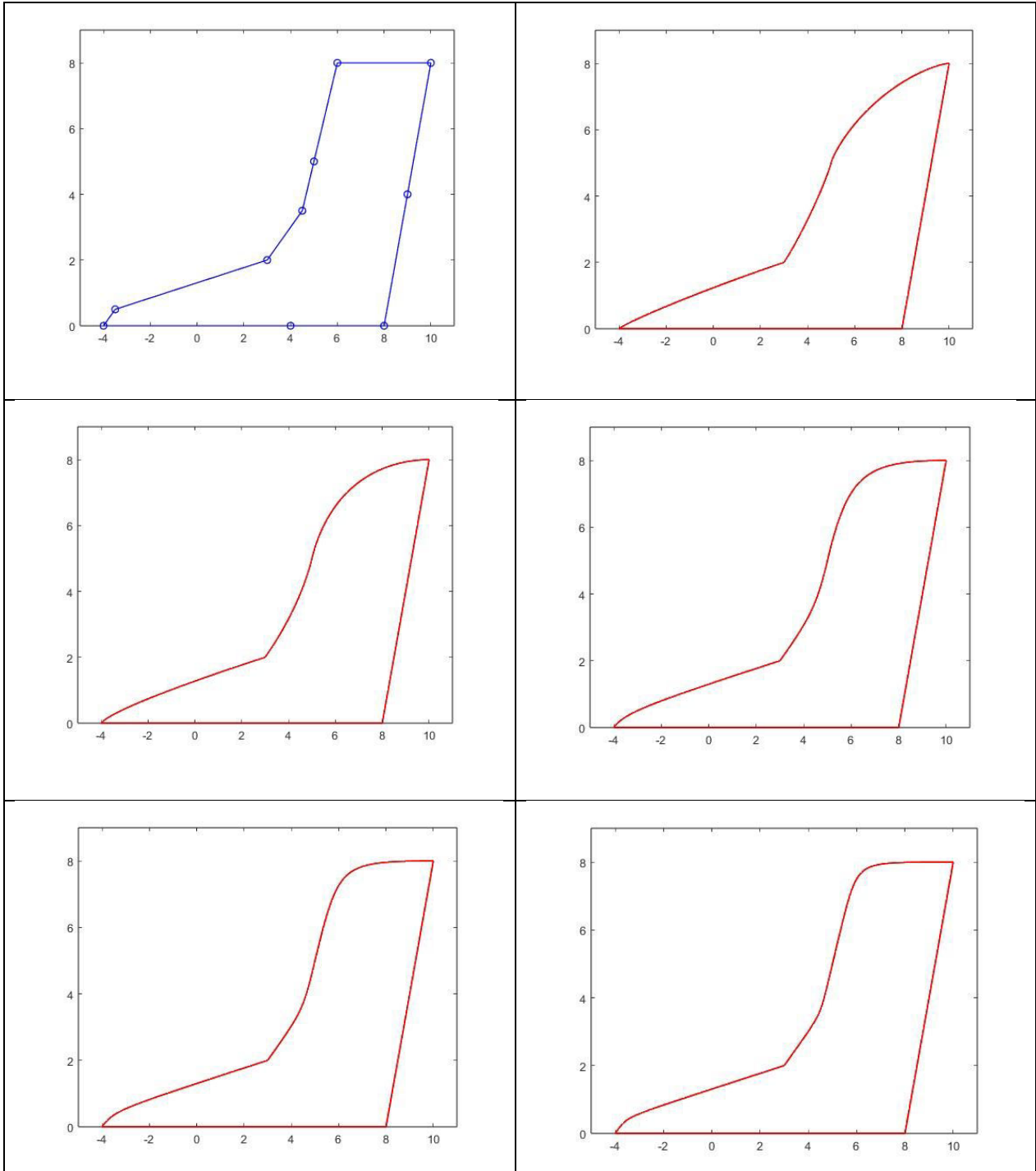


Figure 5: Tail fins of Airplane (Specially Jet Fighter) constructed by Quadratic Hybrid ATE Bézier Curves with different values of shape parameter  $\lambda = -2, -1, 0, 0.5, 1$ .

### V. THE QUADRATIC HYBRID ATE BÉZIER SURFACES

Given control points  $P_{ij}$  ( $i = 0, 1, 2; j = 0, 1, 2$ ) using the tensor product method, we can construct the Quadratic Hybrid ATE Bézier surface

$$T(u, v) = \sum_{i=0}^2 \sum_{j=0}^2 b_{i,2}(\lambda_1, u) b_{j,2}(\lambda_2, v) P_{rs}; \quad (u, v) \in [0, 1] \times [0, 1]$$

where  $b_{i,2}(\lambda_1, u)$  and  $b_{j,2}(\lambda_2, v)$  are Quadratic Hybrid ATE Bézier basis functions. Obviously these surfaces have properties similar to the corresponding Quadratic algebraic Bézier. An example of Quadratic Hybrid ATE Bézier surface is presented in Figure 7.

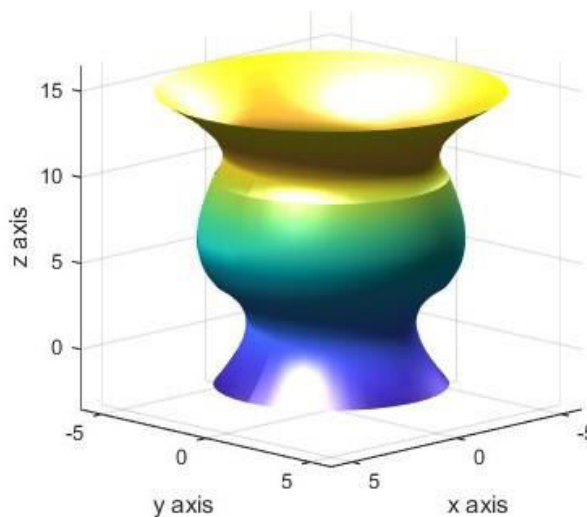


Figure 7: An example of Quadratic Hybrid ATE Bézier surface

### VI. CONCLUSION

As mentioned above Quadratic Hybrid ATE Bézier curve have all the geometric properties that classical quartic Bézier curves have. The shape of the curve can be flexibly controlled by the shape parameter without altering the control points. Since there is nearly no difference in structure between a Quadratic Hybrid ATE Bézier curve and a classical Quadratic Bézier curve, it is not difficult to adapt a Quadratic Hybrid ATE Bézier curve to a CAD/CAM system that already uses the classical quartic Bézier curves. Quadratic Hybrid ATE Bézier curves and surfaces can be used for optimal designing of various parts of Airplane.

### VII. ACKNOWLEDGMENT

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