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Finite Element Quadrilateral Meshing of Quintic to Decic Order

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ABSTRACT: A mesh generation technique for Lagrange family for quintic to decic order quadrilaterals is discussed. The cases related to an equilateral triangle, rectangles, convex polygons, arbitrary oriented rectangles, arbitrary quadrilateral nonconvex polygons is discussed. The mesh is obtained by quadrangulating the given domain. This gives quadrilateral meshes for polygonal domains which are linear in nature. These will be useful to the boundary value problems as an application.

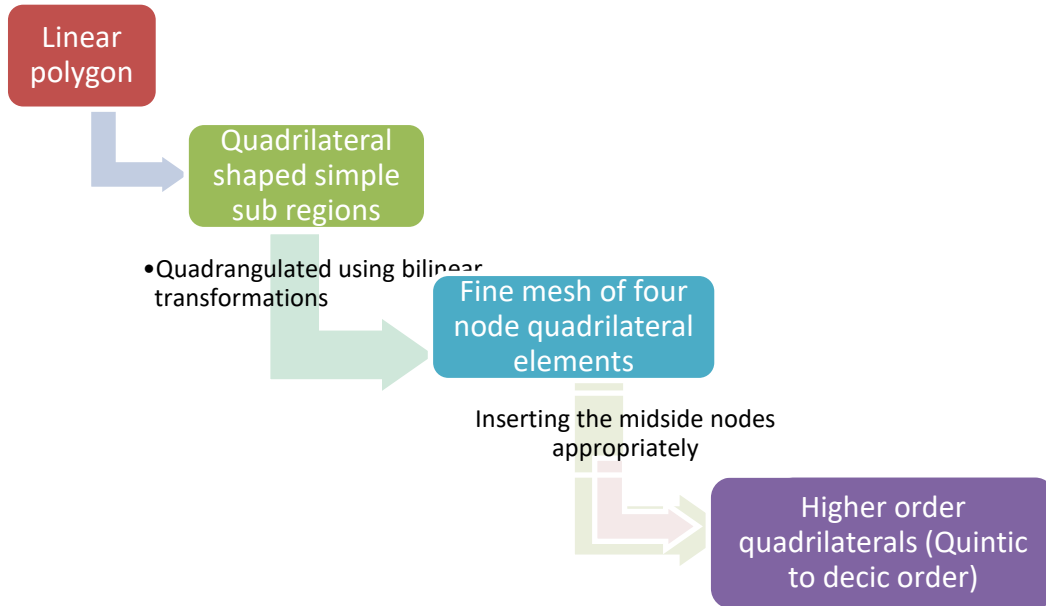
KEYWORDS: Finite element, meshing, quintic, decic, polynomial domain, discretisation.

I. INTRODUCTION

The meshing will be integral part of most of applications. The mesh is obtained by quadrangulating the given domain. This gives quadrilateral meshes for polygonal domains which are linear in nature. Quadrilateral shaped sub regions are obtained initially from the polygonal domain which is linear. Using BLT these sub regions are quadrangulated to an acceptable network of 4- node quadrilateral elements. With the use of mid side nodes suitably the four noded quadrilateral is renovated to quadrilaterals of higher order. The quintic to decic order elements considered for linear polygonal domains. A linear polygon is discretized to simple quadrilateral sub regions. Transformations necessary for generating the degenerate quadrilateral is considered. Higher order quadrilateral is derived by inserting mid side nodes. Later all these combined to get the outcome.

II. LINEAR CONVEX QUADRILATERAL: TRANSFORMATION & MESH GENERATION

Let us contemplate in Cartesian space - (x, y) a random 4- noded linear convex quadrilateral (LCQ) element Q_e . Let the global coordinates of vertices of Q_e be (x_{n_a}, y_{n_a}) . Also choose in local parametric space - (ξ, η) a four noded 2-square, where $(n_a, a = 1, 2, 3, 4)$ are global node numbers. Q_e is mapped to a 2-square mentioned above using isoparametric transformation. The figures Fig.1a and Fig.1b demonstrates this transformation. $\begin{pmatrix} x^e \\ y^e \end{pmatrix} = \sum_{k=1}^4 \frac{1}{4} (1 + \xi \xi_a) (1 + \eta \eta_a) \begin{pmatrix} x_{n_a}^e \\ y_{n_a}^e \end{pmatrix}$ is the Isoparametric coordinate transformation from (x, y) to the (ξ, η) , with vertices of Q_e as $\begin{pmatrix} x_{n_a}^e \\ y_{n_a}^e \end{pmatrix}, a = 1, 2, 3, 4$ and the coordinates of vertices of the 2-square in (ξ, η) space will be $(n_a(\xi_a, \eta_a), a = 1, 2, 3, 4) = \{n_1(-1, -1), n_3(1, 1), n_2(1, -1), n_4(-1, 1)\}$. Taking suitable values for n_1, n_2, n_3, n_4 we can generate various quadrilateral shapes pronounced by the above transformation. The nature of these will control the element geometry. For instance, when $a_1^e = a^e, a_2^e = 0, a_3^e = 0; b_1^e = 0, b_2^e = b^e, b_3^e = 0$ and (a_0^e, b_0^e) be the centroids of the element of triangle then, rectangular elemental structures, that having sides are parallel to coordinate axes were created. The sides of these rectangular structures will be $a^e =$ half side length and $b^e =$ half side width. This makes us to write $x^e = a_0^e + a^e \xi, y^e = b_0^e + b^e \eta$. Further When $a_3^e = 0, b_1^e = 0, b_3^e = 0$, gives $x^e = a_0^e + a_1^e \xi + a_2^e \eta, y^e = b_0^e + b_2^e \eta$. From these relations a parallelogram with two sides are parallel to y-axis will be generated.



THE FLOW CHART OF THE PROCESS

The constants in the transformation will determine the element geometry like rectangular, parallelogram elements etc. Mesh generation for the four-node element is as shown below other examples discussed in applications. An arbitrary linear convex quadrilateral degenerates to give various shapes. The process will cover all bilinear mappings that are necessary for the degenerated cases. An arbitrary quadrilateral Q_e A unit square in local space (u, v) is the image of cartesian space (x, y) , as it's a expedient choice for reduced rectangles or square division. Then unit square into $m \times n$ rectangles. Fig.2a and Fig.2b in (u, v) space, demonstrates, splitting of a unit square: In Fig.2a: an arbitrary quadrilateral Q_e in (x, y) space is divided in to $m \times n$ divisions. [e] in the interior of an element indicates a characteristic quadrilateral element. The quadrilateral nodal values are $\langle n_1, n_2, n_3, n_4 \rangle$, $n_1 = (q - 1)(r + 1) + p$, $n_2 = (q - 1)(r + 1) + p + 1$, $n_3 = q(r + 1) + p + 1$, $n_4 = q(r + 1) + p$. Fig.2b shows the splitting of a unit square in (u, v) space. This unit square is split into four node rectangles with 'mn' in number, with [e]::element'e', [Re]: will be the rectangle with nodal vertices, $(n_a, a=1,2,3,4)$:: local coordinates of unit square are: $n_1 = (q - 1)(r + 1) + p$, $n_3 = q(r + 1) + p + 1$, $n_2 = (q - 1)(r + 1) + p + 1$, $n_4 = q(r + 1) + p$. In addition to this element vertex node numbers are, $((u_k, v_l), k = 1, 2, 3, \dots, n + 1), l = 1, 2, 3, \dots, m + 1)$.

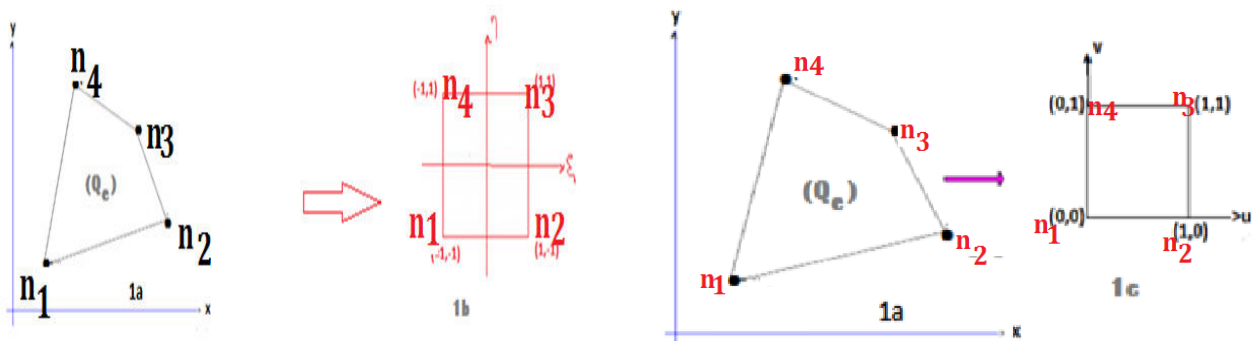
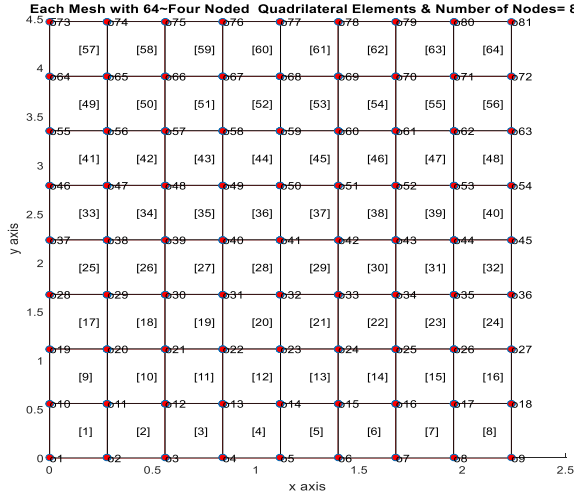
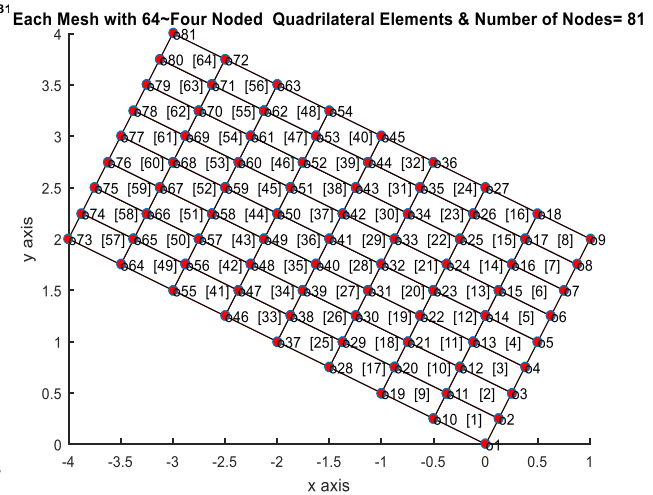


FIG.1A AND 1B A LCQ Q_e AND A STANDARD 2-SQUARE RESPECTIVELY IN (X, Y) SPACE AND (ξ, η) space.

FIG.1A AND 1C: A LCQ AND STANDARD 1-SQUARE RESPECTIVELY IN (X, Y) AND (U, V) SPACE



MESH GENERATION OF A STANDARD SQUARE/RECTANGLE ELEMENT



MESH GENERATION OF A ARBITRARILY ORIENTED SQUARE/RECTANGLE ELEMENT

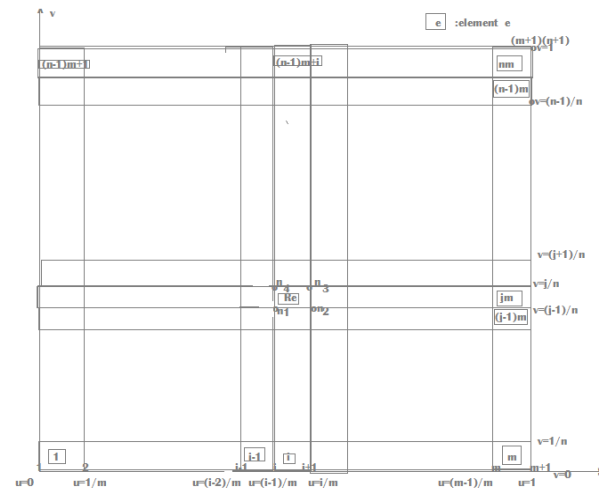
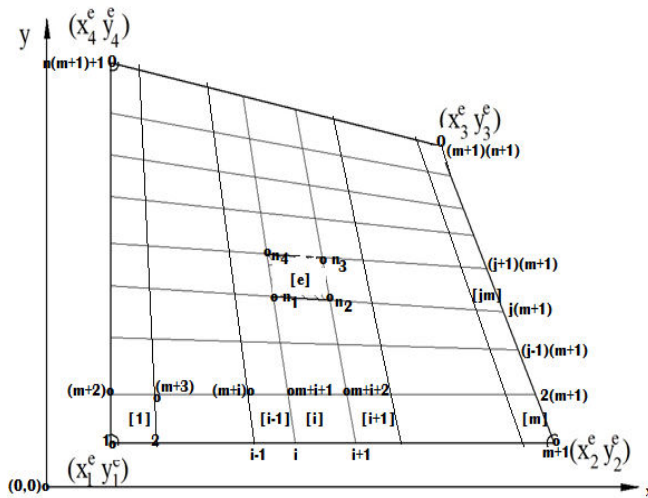


FIG.2A AND 2B : MXN DIVISIONS OF Q_e AND A UNIT SQUARE RESPECTIVELY IN (x,y) SPACE AND (U,V) SPACE

All the coordinates and element nodes can be attained by changing α and β , $\alpha/\beta = 1, 2, \dots, (i + 1)/(j + 1)$ and naturally over any typical element $(\beta + 1) \leq (j + 1)$ and $(\alpha + 1) \leq (i + 1)$. The succeeding figure depicts node numbering with naming interchanged i, j with α, β respectively:

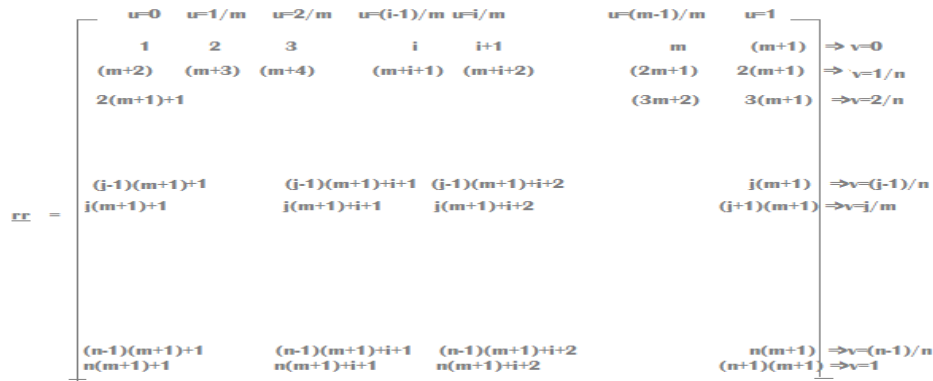
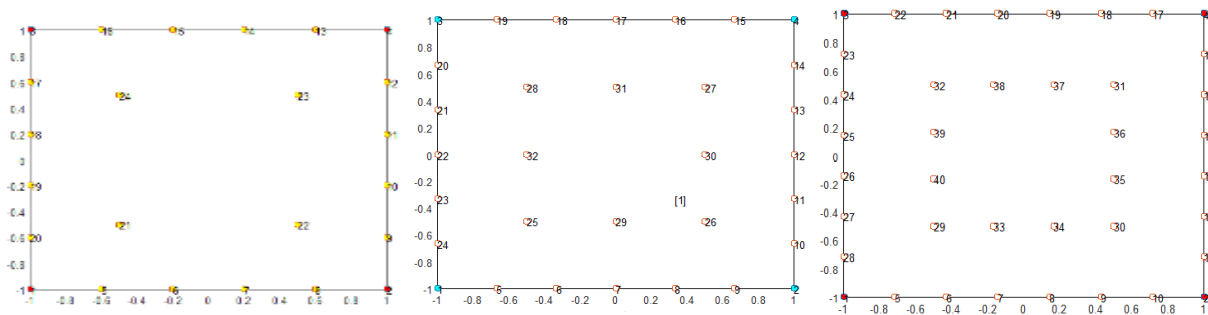
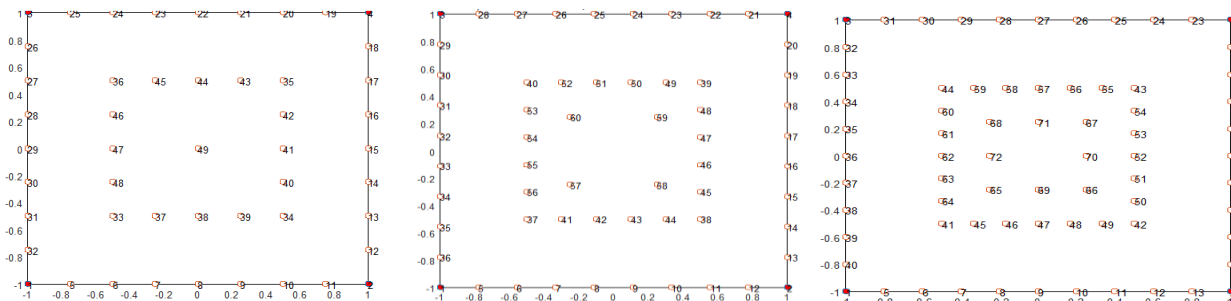


FIG 2C. THE DIVISION OF A UNIT SQUARE;S NODE NUMBERS MATRIX REPRESENTED AS RR

The corners of Q_e and the unit square is considered as $n_1 = 1$, $n_3 = (m + 1)(n + 1)$ and $n_4 = n(m + 1) + 1$, where the number of dissections along x/u axis is denoted by i and j denotes the numeral of dissections along y/v axis in both the coordinate system spaces. The boundary nodes of a single quadrilateral ($m=1, n=1$) are $n_1 = 1$, $n_3 = 4$, $n_2 = 2$, and $n_4 = 3$. This will be useful in the explicit demonstration of complete Lagrange elements with quintic to decic order over the standard squares. The Serendipity, Lagrange quadratic and cubic elements also considered. Finite element interconnects for quintic order to decic order complete Lagrange elements is obtainable. The succeeding figures shows Complete Lagrange family rectangular elements. We have shown over a 2-square from Quintic to Decic order elements.

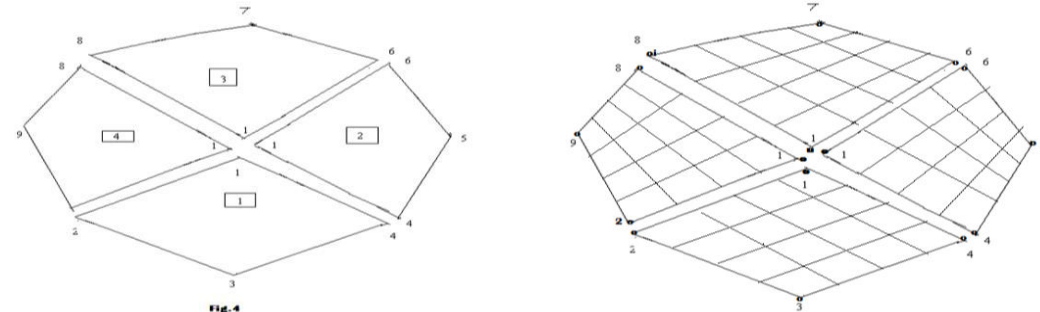


Lagrange family rectangular elements over a 2-square from Quintic to Decic order.



Lagrange family rectangular elements over a 2-square from Quintic to Decic order.

For a Quadrilateral, meshes is obtained by dividing square of side 1 unit into ‘mn’ squares or rectangles of uniform dimensions, with ‘(m+1)*(n+1)’ nodes. Then using bilinear transformation nodes in uv-space is obtained that creates ‘mn’ rectangles uv-space. Then mid side nodes and inside nodes if any is inserted. We will obtain higher order quadrilaterals from these ‘mn’ number of quadrilaterals. Meshes for polygonal domains are obtained by apportioning territory into an abrasive mesh of quadrilaterals or triangles or both, then restoring together.



DEGENERATION OF A CONVEX POLYGON

RESTORING OF FOUR QUADRILATERALS TO FORM A CONVEX POLYGON

III. CONCLUSIONS

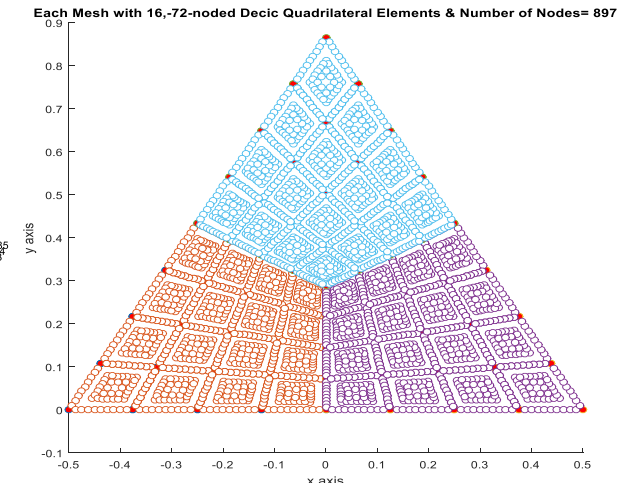
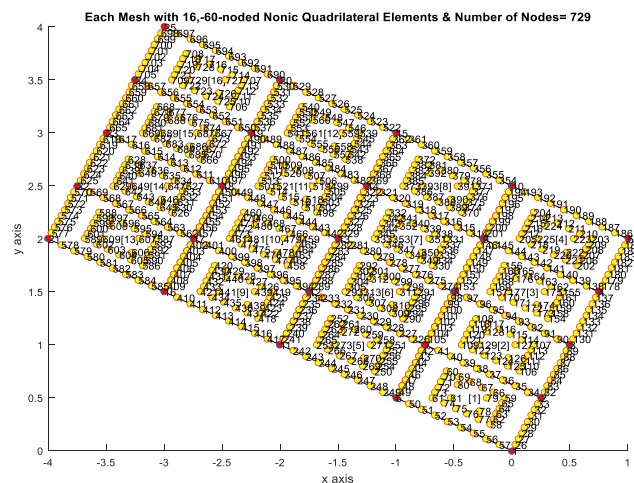
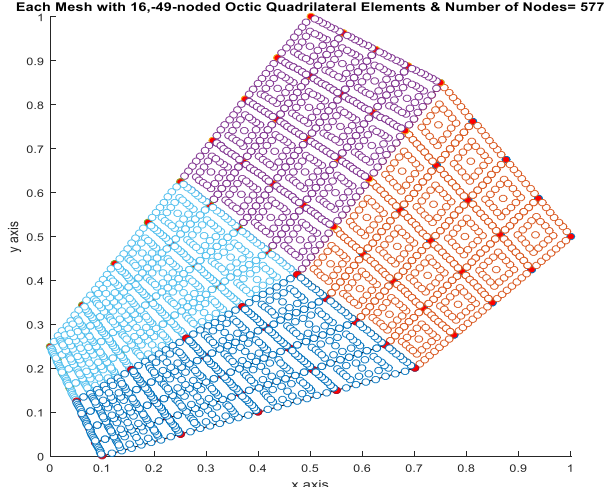
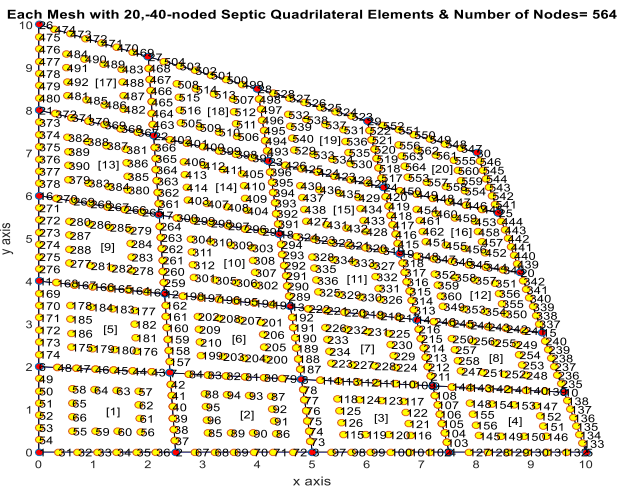
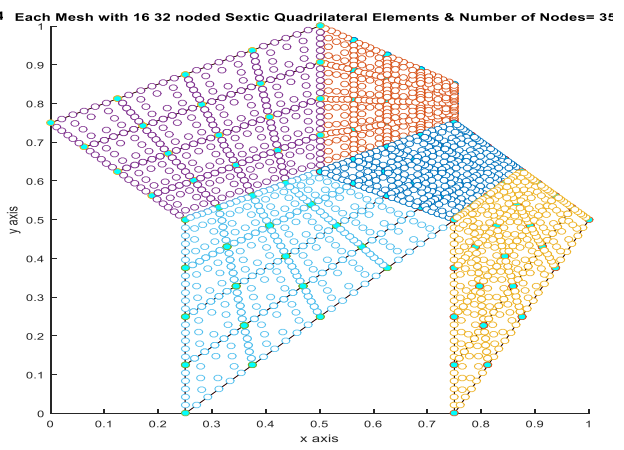
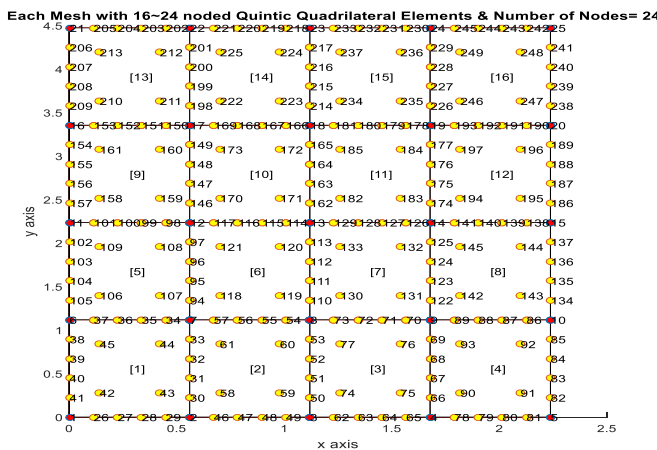
In some problems of boundary value in nature, the discretization of some polygonal structure with linear, cubic, quadratic, quartic finite elements may be necessary. To facilitate such problems here we have demonstrated a mesh generation scheme for linear polygonal domains in terms of all quadrilateral, quintic to decic order quadrilaterals elements. The aim is to have codes that automatically give element over linear convex quadrilateral domain taking the coordinates of the vertices as input. Mesh generation with Finite Elements with the help of Quintic to Decic order elements for an Equilateral Triangle, an Arbitrary Quadrilateral, A Nonconvex Polygon, A Convex Polygon are obtained.

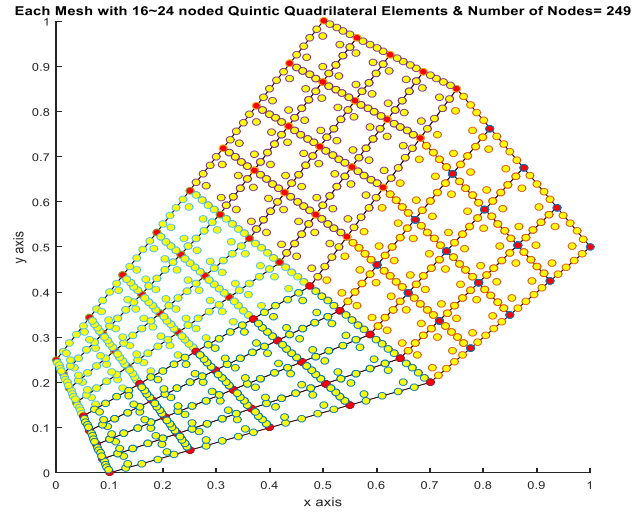
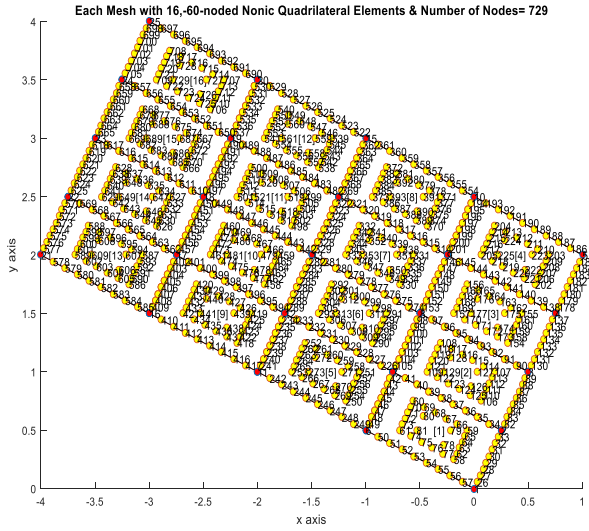
REFERENCES

1. Peraire. J. Vahdati. M, Morgan. K and Zienkiewicz. O. C, Adaptive remeshing for compressible flow computations, J. Comp. Phys. 72, 449-466(1987)
2. George. P. L, Seveno. E, The advancing-front mesh generation method revisited. Int. J. Numer. Meth. Eng 37, 3605-3619(1994)
3. Masud. A, Khurram. R. A, A multiscale/stabilized finite element method for the advection-diffusion equation, Comput. Methods. Appl. Mech. Eng 193, 1997-2018(2004)
4. Johnston. B.P, Sullivan. J. M and Kwasnik. A, Automatic conversion of triangular finite element meshes to quadrilateral elements, Int. J. Numer. Meth. Eng. 31, 67-84(1991)
5. Zhu. J, Zienkiewicz. O. C, Hinton. E, Wu. J, A new approach to the development of automatic quadrilateral mesh generation, Int. J. Numer. Meth. Eng 32(4), 849-866(1991)
6. Lau. T. S, Lo. S. H and Lee. C. S, Generation of quadrilateral mesh over analytical curved surfaces, Finite Elements in Analysis and Design, 27, 251-272(1997)
7. Park. C, Noh. J. S, Jang. I. S and Kang. J. M, A new automated scheme of quadrilateral mesh generation for randomly distributed line constraints, Computer Aided Design 39, 258-267(2007)
8. H.T. Rathod, K. Sugantha devi, An All Quadrilateral Automesh Generation Technique and Explicit Integration Scheme for Finite Elements to Solve Some Elliptic Boundary Value Problems in Two Space, International Journal of Engineering and Computer Science, ISSN 2319-7242, Vol.5 issue 12(December 2016), pp 19674-19778
9. H.T.Rathod, Bharath Rathod, Sugantha Devi.K, Hariprasad.A.S, Finite element solution of Poisson Equation over Polygonal Domains using a novel auto mesh generation technique and an explicit integration scheme for eight node linear convex quadrilaterals, International Journal of Engineering and Computer Science, 6 (10), 2017, 22632-22787.
10. H. T. Rathod, K.V.Vijayakumar, A. S. Hariprasad , Gauss Legendre Quadrature Formulas for Polygons Using an All Quadrilateral Finite Element Mesh of Triangular Surfaces, International Journal of Engineering and Computer Science, 3 (6), 2014, 6636-6680.
11. H. T. Rathod, Md.Shafiqul Islam. H. Y. Shrivall , Bharath Rathod, K. Sugantha Devi, Finite element solution of Poisson Equation over Polygonal Domains using a novel auto mesh generation technique and an explicit integration scheme for nine node linear convex quadrilateral of Lagrange family. International Journal Of Engineering And Computer Science, 6(11), 2017, 22869-23058.
12. M.Okabe, Complete Lagrange family for the cube in Finite Element interpolations, Comput. Meth. Appl. Mech. Engng.29(1981)51-56.

13. H.T, Rathod and Sridevi.Kilari, General complete Lagrange family for the cube in finite element interpolations, Comput.Meth.Appl.Mech and Engrg 181(2000)295-344.
14. Kallur V Vijayakumar, A S Hariprasad. "Curved domain discretization with boundary defined by polar equations", Journal of Physics:Conference Series, 1473 (2020).

FIGURES







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