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Nonlinear Optimization Approach for Solving the DC-OPF Problem using Matlab fmincon Function

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ABSTRACT: This paper presents a novel approach for solving the DC Optimal Power Flow (DC-OPF) problem from a nonlinear optimization perspective, offering a flexible, accurate, and easily implementable solution using Matlab-constrained nonlinear optimization function fmincon. The proposed method involves formulating the AC Optimal Power Flow (AC-OPF) problem and subsequently transforming it into a DC-OPF model by integrating and modifying specific constraints. This results in a DC-OPF model capable of handling nonlinear optimization functions and active power flow equations, eliminating the linear approximation typically used in traditional DC-OPF models. The reliability and accuracy of the proposed method are demonstrated through two case studies using the 5-bus test power system and the 39-bus New England power system.

KEYWORDS: Nonlinear optimization, fmincon function, Matlab, DC-OPF.

I. INTRODUCTION

Power flow analysis is one of the most widely used tools for determining steady-state operating points in power systems. However, this tool does not account for economic, environmental, or system security aspects, making it necessary to develop the Optimal Power Flow model to incorporate these factors into power system operation. The OPF problem is formulated as a constrained, non-convex nonlinear optimization problem, where an objective function is minimized while satisfying equality and inequality constraints [1-3].

In the AC-OPF problem, the objective function is nonlinear and may represent generation costs, CO₂ emissions, system security, or other operational goals. Equality constraints model the active and reactive power balance at each power system node, while inequality constraints represent the system's physical and operational limits, such as nodal voltage magnitudes and active and reactive power generation limits.

Due to its nonlinear optimization nature, solving the AC Optimal Power Flow model is more complex and requires greater computational time and effort, especially when analyzing large-scale power systems. In power system studies, certain analyses require determining multiple steady-state operating points under specific operating conditions to evaluate system behavior and performance. In such cases, using conventional power flow and AC-OPF models becomes impractical due to the significant computational time and effort needed to determine various steady-state operating points.

For this reason, most tools used in these studies rely on linearization and approximation techniques to simplify AC models and bypass nonlinearity. Specifically, in the AC-OPF model, some techniques that help overcome its complexity include small-angle approximations [4] or relaxation methods [5-9]. Typically, AC-OPF nonlinearity is

addressed using a simplified model where power losses are neglected, and active power flow equations are linearized, assuming reactive power throughout the system is zero. This assumption is based on nodal voltages being equal to the system's base voltage.

This simplification leads to a convex linear optimization problem known as DC Optimal Power Flow, where the variables are real power generation and the nodal phase angle of the power system [10, 11]. DC-OPF analysis is widely used in Mexico and globally for transmission planning, market price calculation, reliability assessment, contingency analysis, and energy adequacy studies [11, 12]. Its convex nature and computational advantages allow for fast-scale resolution [13].

The DC Optimal Power Flow problem can be solved using Linear Programming (LP) or Quadratic Programming (QP) techniques [14]. In the case of the LP-based DC-OPF model, both the system constraints and the objective function must be linear, as nonlinear formulations do not meet the requirements of linear programming [15]. On the other hand, QP is a special case of nonlinear programming where the objective function is quadratic, while system constraints remain linear.

Considering these aspects, this work employs Matlab-constrained nonlinear optimization function `fmincon`, as it can use Sequential Quadratic Programming (SQP) to solve nonlinear optimization problems [16]. Without using `fmincon`, implementing a linear optimization algorithm would require linearizing both the objective function and active power flow equations, followed by solving the linear optimization model using an appropriate optimization method.

In this context, the optimization algorithm implemented in this study, leveraging the `fmincon` function, simplifies the development process and reduces computation time for solving the DC-OPF problem. Additionally, `fmincon` provides flexibility for adapting and extending the proposed approach to conduct other types of analyses commonly used in power system studies.

The structure of the paper is as follows: Section II presents the general formulations of the DC and AC Optimal Power Flow problems, along with an approximate method for estimating active power losses in the power system's transmission elements. The proposed solution for solving the DC-OPF model using the nonlinear optimization function `fmincon` is described in Section III. Two case studies involving two power systems are presented and analyzed in detail in Section IV, demonstrating the reliability and accuracy of the proposed solution. Finally, the conclusions are summarized in Section V

II. DC-OPF AND AC-OPF FORMULATION

A. AC-OPF Formulation

The AC Optimal Power Flow problem involves minimizing a specific objective function, typically the active power generation cost, while satisfying a set of physical and operational constraints of the electric power system. These constraints are modeled through equality and inequality conditions within the OPF formulation. The equality constraints represent the balance of active and reactive power at each bus in the power system, while the inequality constraints define operational limits for buses and generators. The general formulation of the AC-OPF problem is as follows:

$$\text{Minimizar } C_T = \sum_{i=1}^{ng} (a_i + b_i P_{gi} + c_i P_{gi}^2) \quad (1)$$

$$\text{Sujeta a } P_{gi} - P_{di-} - \sum_{i=1}^{ng} P_{iny,i-j} = 0 \quad (2)$$

$$Q_{gi} - Q_{di-} - \sum_{i=1}^{ng} Q_{iny,i-j} = 0 \quad (3)$$

$$\delta_i^{\min} \leq \delta_i \leq \delta_i^{\max} \quad (4)$$

$$V_i^{\min} \leq V_i \leq V_i^{\max} \quad (5)$$

$$P_{gi}^{\min} \leq P_{gi} \leq P_{gi}^{\max} \quad (6)$$

$$Q_{gi}^{\min} \leq Q_{gi} \leq Q_{gi}^{\max} \quad (7)$$

where C_T represents the total system cost and the objective function to be minimized. The equality constraints given by Eqs. (2) and (3) model the active and reactive power balance, while the inequality constraints (4)–(7) represent the system's physical and operational limits.

B. DC-OPF Formulation

The DC Optimal Power Flow problem simplifies the AC-OPF model by linearizing the power flow equations and the objective function. Similar to the AC-OPF, the DC-OPF aims to minimize the active power generation cost while satisfying the system's physical and operational constraints. However, the number of variables is reduced, considering only generator power output and bus phase angles. In this formulation, reactive power is assumed negligible, and bus voltages are set equal to the system's nominal or base voltage, typically 1.0 p.u., significantly simplifying the OPF problem. Thus, the general formulation of the DC OPF problem is expressed as follows [17]:

$$\text{Minimize } C_T = \sum_{i=1}^{ng} (b_i P_{gi}) \quad (8)$$

$$\text{Subject to } P_{gi} - P_{di-} - \sum_{i=1}^{nl} F_{ij} = 0 \quad (9)$$

$$F_{ij} = \frac{\delta_i - \delta_j}{x_{ij}} \quad (10)$$

$$\delta_i^{ref} = 0 \quad (11)$$

$$\delta_i^{\min} \leq \delta_i \leq \delta_i^{\max} \quad (12)$$

$$F_{ij}^{\min} \leq F_{ij} \leq F_{ij}^{\max} \quad (13)$$

$$P_{gi}^{\min} \leq P_{gi} \leq P_{gi}^{\max} \quad (14)$$

In the preceding equations, F_{ij} represents the active power flow through transmission elements, either lines or transformers, expressed in direct current. It is important to note that the objective function given by Eq. (8) considers only the linear part of the cost function commonly used in the AC-OPF model. Alternatively, the objective function can be handled using a piecewise linear approximation, enabling a fully linear DC-OPF optimization model [15].

III. NONLINEAR OPTIMIZATION APPROACH FOR THE DC-OPF PROBLEM

The proposed approach for solving the DC-OPF model involves first formulating an AC-OPF using Matlab constrained nonlinear optimization toolbox, specifically the `fmincon` optimization function. The AC-OPF model is then converted to a DC-OPF by integrating certain constraints and modifications. As mentioned in the previous section, the DC-OPF formulation assumes all buses operate at the base voltage, fixed at 1.0 p.u. Consequently, equality and inequality constraints associated with reactive power are not considered, as reactive power depends on voltage magnitude.

Thus, the equality constraint given by Eq. (3) and the inequality constraint in Eq. (7) are omitted when transitioning from AC-OPF to DC-OPF. Additionally, the inequality constraint in Eq. (5), which specifies bus voltage limits, is reformulated as an equality constraint that penalizes bus voltages deviating from the base voltage, fixing the voltage V_i at 1.0 pu, as shown in Eq. (18). This modification yields a model capable of addressing the DC-OPF problem while considering both the nonlinear cost objective function and the nonlinear expression of active power flow equations.

$$\text{Minimize } C_T = \sum_{i=1}^{ng} (a_i + b_i P_{gi} + c_i P_{gi}^2) \quad (15)$$

$$\text{Sujeta a } P_{gi} - P_{di} - \sum_{i=1}^{ng} P_{iny,i-j} = 0 \tag{16}$$

$$\delta_i^{\min} \leq \delta_i \leq \delta_i^{\max} \tag{17}$$

$$V_i = V_{Base} = 1.0 \text{ pu} \tag{18}$$

$$P_{gi}^{\min} \leq P_{gi} \leq P_{gi}^{\max} \tag{19}$$

It is important to note that Eq. (16) in the proposed nonlinear approach is identical to Eq. (2) from the AC-OPF model. This implies that, in the proposed DC-OPF solution, it is unnecessary to simplify the power flow equations traditionally linearized in conventional DC models. This implicit linearization occurs by integrating the previously mentioned constraints into the model. The schematic in Fig. 1 illustrates the process of deriving the proposed DC-OPF model from the AC-OPF formulation.

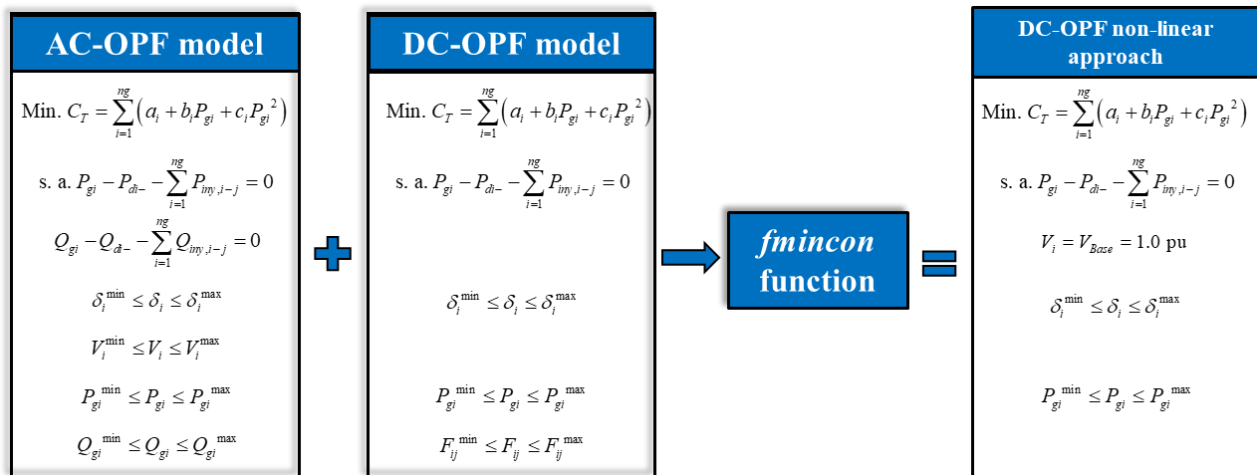


Fig. 1. Schematic of conversion from AC-FPO model to DC-FPO model.

It should be emphasized that the proposed modifications yield an optimization problem with a nonlinear objective function and linear constraints. Thus, the model defined by Eqs. (17) to (21) can be effectively solved using Nonlinear Quadratic Programming (NLP). Finally, considering the nonlinear objective function enhances result accuracy by eliminating the need for linear approximations. This approach also reduces computation time by simplifying the implementation of the DC OPF algorithm, thanks to the reduced model complexity.

IV. NUMERICAL EXAMPLES

The purpose of the numerical examples is to demonstrate the reliability and accuracy of the proposed solution for solving the DC OPF problem. For this purpose, the IEEE 5-bus test system and the New England 39-bus system with 10 generators were used, with data sourced from [18] and [19], respectively. The results of the proposed approach were compared with those obtained using MatPower.

As widely known, MatPower is an open-source Matlab-based package commonly used for optimization and steady-state power system simulations, including DC-OPF modeling [20]. For this reason, it was selected as a reference to compare and validate the proposed solution's results. The case studies were conducted with a convergence tolerance of 1×10^{-9} on a desktop computer equipped with an AMD Ryzen 5 5600X six-core processor running at 3.7 GHz and 32 GB of DDR4 RAM at 3600 MHz.

A. IEEE test power system

This section presents the results of the numerical example using the 5-bus power system. As mentioned in Section III, the bus voltage magnitude was constrained by an equality condition, fixed at 1.0 p.u. pu ($|V_i|=|V_B|=1.0 \text{ pu}$), while the initial phase angle was set at 0° , with limits of $0^\circ \leq \delta_i \leq 360^\circ$. The initial active power generation for the two generators

of power system was defined as $P_{gi}=0.0$ p.u., with generation limits of $0 \text{ MW} \leq P_{gi} \leq 200 \text{ MW}$. The cost function coefficients for the two generators considered in the numerical example were $a_i=0.006$ \$/hr, $b_i= 0.034$ \$/hrMW y $c_i= 0.004$ \$/hrMW².

It is important to note that active power losses were not calculated in this numerical example. The primary objective was to compare the proposed solution’s results with those obtained using MatPower, which lacks an integrated algorithm for computing transmission element losses in the DC OPF model. However, the proposed solution includes a computational structure for this calculation.

Considering the parameters and data for the 5-bus power system provided in [18], the steady-state optimal operating point in direct current was obtained in 0.44 seconds using the proposed solution and in 0.48 seconds using MatPower. Table I and Fig. 1 compare the steady-state operating points obtained by both methods. Additionally, Table II presents a comparison of power flows computed by the two approaches.

Table 1. Steady-state operating point comparison for the 5-node power system.

Node	Proposed approach	MatPower
1	0.000°	0.000°
2	-1.706°	-1.705°
3	-5.485°	-5.482°
4	-4.787°	-4.785°
5	-4.526°	-4.523°

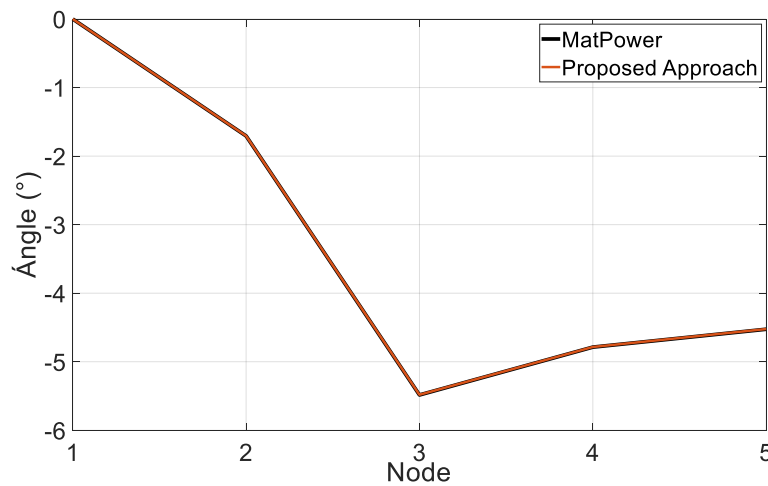


Fig. 2. Graphical comparison of the steady-state operating point.

Table 2. Results of active power flows obtained by the proposed solution and Matpower.

Lines		Power flow (MW)	
Sending node	Reception node	Approach proposed	MatPower
1	2	49.62	49.61
1	5	32.88	32.89
2	5	27.33	27.32
2	4	29.86	29.86

2	3	54.93	54.93
5	4	15.21	15.21
4	3	5.07	5.07

It should be mentioned that Table II does not show the active power flow data from the receiving node to the sending node, since, as there are no power losses, the active power flow being sent is the same as the power flow being received, but with opposite sign. The results of the phase angles show that the nodal angle profile is very similar in both cases, having variation of thousandths between them. Regarding the power flows, it is also observed that the results obtained with the proposed solution are very accurate concerning those obtained in MatPower. Finally, Table III shows a summary of the results of the DC FPO for the 5-node system, where the solution obtained with the proposed method and the one calculated by MatPower are compared.

Table 3. Summary of results for the 5-node power system.

Parameter	Approach proposed	MatPower
Total power generation	165 MW	165 MW
Generation Cost	735.450 \$/hr	735.450 \$/hr

As with the previous data, the generated power and generation cost obtained using the proposed solution are very similar to those obtained using MatPower. It is not omitted to mention that the active power losses in the transmission elements of the 5-node power system were 6.742 MW, increasing active power generation, giving a P_g equal to 167.997 MW and hence an increase in generation cost of \$747,756/hr. As mentioned before, these data of active power losses are not compared with MatPower.

B. New England power system

In this numerical example, an equivalent system of the New England power system is used. Similar to the previous numerical example, the nodal voltage magnitude was set to 1.0 pu ($|V_i|=|V_B|=1.0$ pu) and the initial condition of the phase angle was considered to be 0° with limits of $0^\circ \leq \delta_i \leq 360^\circ$. Likewise, the initial condition of the active power generation in all the generators of the system was $P_{gi}=0.0$ pu. The active power generation limits and the coefficients of the cost functions of the generators that were considered for this numerical example are given in Table IV.

Table 4. Generators data for the New England power system.

Gen	P_{gi}^{\min} (MW)	P_{gi}^{\max} (MW)	a_i (\$/hr)	b_i (\$/hrMW)	c_i (\$/hrMW ²)
30	0	350	0.0193	0.069	0.0193
31	0	650	0.0111	0.037	0.0111
32	0	800	0.0104	0.028	0.0104
33	0	750	0.0088	0.047	0.0088
34	0	650	0.0128	0.028	0.0128
35	0	750	0.0094	0.037	0.0094
36	0	750	0.0099	0.048	0.0099
37	0	700	0.0113	0.036	0.0113
38	0	900	0.0071	0.037	0.0071
39	0	1200	0.0064	0.039	0.0064

The computation time used to determine the DC optimal steady-state operating point for this numerical example was 0.875 seconds using the proposed solution and 0.9 seconds using MatPower. A comparative summary of results for the

New England system is presented in Table V, while the operating points corresponding to the DC-OPF solution, obtained using both applications, are presented in Table VI.

Table 5. Summary of results for the New England power system.

Parameter	Approach proposed	MatPower
Total power generation	6,137.30 MW	6,137.30 MW
Generation Cost	60,858.791 \$/hr	60,858.79 \$/hr

As in the previous numerical example, the results of the proposed solution are very similar to those obtained by Matpower, so it is possible to infer that the solution proposed in this work is reliable and accurate to solve the DC Optimal Power Flow problem. The variations in the results between both tools are very small, which makes the results of the proposed solution acceptable to determine optimal steady state operation points.

Table 6. Steady-state operating point comparison for the New England power system.

Bus	Propuesta	MatPower	Bus	Propuesta	MatPower
1	2.191	2.190	21	2.191	2.190
2	5.794	5.792	22	5.794	5.792
3	2.662	2.661	23	2.662	2.661
4	1.482	1.481	24	1.482	1.481
5	2.462	2.461	25	2.462	2.461
6	3.155	3.154	26	3.155	3.154
7	0.864	0.864	27	0.864	0.864
8	0.336	0.336	28	0.336	0.336
9	0.137	0.137	29	0.137	0.137
10	5.708	5.707	30	5.708	5.707
11	4.830	4.829	31	4.830	4.829
12	4.835	4.834	32	4.835	4.834
13	5.028	5.027	33	5.028	5.027
14	3.476	3.474	34	3.476	3.474
15	3.493	3.492	35	3.493	3.492
16	5.225	5.223	36	5.225	5.223
17	4.106	4.105	37	4.106	4.105
18	3.096	3.095	38	3.096	3.095
19	10.652	10.642	39	10.652	10.642
20	9.358	9.349			

It should be noted that the proposed solution can be extended to linearize the models of devices and equipment used in power systems, such as FACTS devices, which can be integrated in the DC-OPF formulation or in other applications where it is necessary to simulate DC power systems, such as contingency analysis, transmission planning, and expansion, among others.

V. CONCLUSIONS

A proposal to solve the DC-OPF problem has been presented. The proposed solution is based on the AC-OPF formulation in which the constraints are linearized and the nonlinear objective function is maintained, resulting in a

Nonlinear Quadratic Programming problem, where it is not necessary to linearize or approximate such objective function. The reliability and accuracy of the proposed solution were demonstrated through two case studies with the 5-node test power system and the 39-node New England system in which the results obtained using the proposed solution and using the MatPower package are compared. The proposed solution using the fmincon function is flexible and easy to implement, therefore, it is possible to extend its capabilities to perform studies where the application of Quadratic Programming is necessary. In this sense, the solution proposed here has the potential to be used for educational and even research purposes, since it offers the advantage that the models of other elements of the power system, such as FACTS devices, can be easily linearized to be included in reliability analysis, power markets or other studies where the application of the FPO model in DC is involved.

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