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# Lucas Antimagic Labeling of Some Generalized Graphs and its Subdivision

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**ABSTRACT:** A  $(p, q)$  graph  $G$  is said to be a Lucas antimagic graph if there exists a bijection  $f: E(G) \rightarrow \{L_1, L_2, \dots, L_q\}$  such that the induced injective function  $f^*: V(G) \rightarrow \{1, 2, \dots, \sum L_q\}$  given by  $f^*(u) = \sum_{e \in E(u)} f(e)$  are all distinct (where  $E(u)$  is the set of edges incident to  $u$ ).

In this paper the Lucas Antimagic Labeling of some generalized graphs and its subdivision are found.

**KEYWORDS:** Petersen graph, Complete Bipartite graph, Lucas Antimagic graph.

## I. INTRODUCTION

In this study, we explore graph  $G(V, E)$  characterized as finite, simple, and undirected with  $p$  vertices and  $q$  edges. Graph labeling stands as a cornerstone concept in graph theory, involving the assignment of integers to vertices or edges. This concept's vast applications across fields such as astronomy, coding theory, and beyond, have brought it to the forefront of research.

The journey into this intriguing research domain was profoundly inspired by the seminal contributions of Gallian, whose comprehensive survey [1] serves as a cornerstone in the field. Gallian's work meticulously catalogues numerous labeling methods and their diverse applications, providing a robust foundation upon which we have embarked on our own research endeavors.

A pivotal and intriguing concept that particularly captivated our attention is Antimagic labeling, introduced by the pioneering efforts of N. Hartsfield and G. Ringel in the year 1990. This innovative labeling method has sparked a surge of interest and subsequent investigations, opening new vistas for exploration within the rich landscape of graph theory. Antimagic labeling, with its unique properties and potential applications, offers a fertile ground for further inquiry and discovery.

Inspired by these groundbreaking contributions, we have introduced the novel concept of Lucas Antimagic labeling. This labeling scheme builds upon the principles of Antimagic labeling, integrating the properties of Lucas numbers to create a distinctive and versatile labeling method. Our research delves into the intricacies of Lucas Antimagic labeling, exploring its applicability on generalized Petersen graph and complete bipartite graph and their subdivisions.

## II. DEFINITIONS

**Definition 2.1:[2]** Lucas number is defined by the linear recurrence relation

$$L_1 = 2, L_2 = 1 \text{ and } L_n = L_{n-1} + L_{n-2}, n > 2$$

The first few Lucas numbers are 2, 1, 3, 4, 7, 11, 18, 29, 47, ...

**Definition 2.2:[2]** A  $(p, q)$  graph  $G$  is said to be a Lucas antimagic graph if there exists a bijection  $f: E(G) \rightarrow \{L_1, L_2, \dots, L_q\}$  such that the induced injective function  $f^*: V(G) \rightarrow \{1, 2, \dots, \sum L_q\}$  given by  $f^*(u) = \sum_{e \in E(u)} f(e)$  are all distinct (where  $E(u)$  is the set of edges incident to  $u$ ).

**Definition 2.3:[3]** The Generalized Petersen graph  $Gp(n, k)$  with  $n \geq 3$  and  $1 \leq k \leq \lfloor n/2 \rfloor$  are defined to be a graph with  $V(Gp(n, k)) = \{v_i, u_i : 0 \leq i \leq n - 1\}$ ,  $E(Gp(n, k)) = \{v_i v_{i+1}, u_i u_{i+k}, v_i u_i : 0 \leq i \leq n - 1\}$  Where the subscripts are taken modulo  $n$ .

**Definition 2.4:[1]** A complete bipartite graph is a special type of bipartite graph where every vertex of one set is connected to every vertex of the other set.

### III. MAIN RESULTS

**Theorem 3.1:**

The Generalized Petersen graph  $Gp(n, k)$  ( $n \geq 3$ ) is a Lucas antimagic graph.

Proof:

Let  $G$  be the Generalized Petersen graph  $Gp(n, k)$  with ( $n \geq 3$ ) and  $k < \frac{n}{2}$  defined to be a graph with  $V(G) = \{v_i, u_i : 0 \leq i \leq n - 1\}$

$$E(G) = \{v_i v_{i+1}, u_i u_{i+k}, v_i u_i : 0 \leq i \leq n - 1\}$$

Where the subscripts are taken modulo  $n$ .

Define a function  $f: E(G) \rightarrow \{L_1, L_2, \dots, L_q\}$  by

$$f(v_i v_{i+1}) = L_{i+1}, 0 \leq i \leq n - 1$$

$$f(v_i u_i) = L_{n+1+i}, 0 \leq i \leq n - 1$$

$$f(u_i u_{i+k}) = L_{2n+1+i}, 0 \leq i \leq n - 1$$

The induced function  $f^*: V(G) \rightarrow \{1, 2, \dots, \sum L_q\}$  is given by

$$f^*(v_0) = L_1 + L_n + L_{n+1}$$

$$f^*(v_i) = L_i + L_{i+1} + L_{n+1+i}, 1 \leq i \leq n - 1$$

$$f^*(u_0) = L_{2n+1} + L_{3n} + L_{n+1}$$

$$f^*(u_i) = L_{n+1+i} + L_{2n+i} + L_{2n+1+i}, 1 \leq i \leq n - 1$$

We observe that the vertices are all distinct.

Hence  $G$  is a Lucas antimagic graph.

**Example 3.2:** The Lucas antimagic labeling of the generalized Petersen graph  $Gp(n, k)$  is shown below.

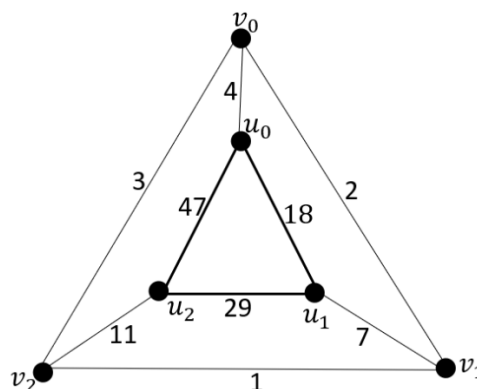


Figure 1 : Generalized Petersen graph  $Gp(3, 1)$

**Theorem 3.3:**

The subdivision graph  $S(Gp(n, k))$  ( $n \geq 3$ ) is a Lucas antimagic graph.

Proof:

Let  $G$  be  $S(Gp(n, k))$ .

$V(G) = \{v_i, u_i, a_i, b_i, w_i : 0 \leq i \leq n - 1\}$   
 $E(G) = \{v_i b_i, b_i v_{i+1}, u_i a_i, a_i u_{i+1}, v_i w_i, u_i w_i : 0 \leq i \leq n - 1\}$   
 Where the subscripts are taken modulo  $n$ .  
 Define a function  $f: E(G) \rightarrow \{L_1, L_2, \dots, L_q\}$  by  
 $f(v_i b_i) = L_{i+1}, 0 \leq i \leq n - 1$   
 $f(b_i v_{i+1}) = L_{n+1+i}, 0 \leq i \leq n - 1$   
 $f(u_i a_i) = L_{2n+1+i}, 0 \leq i \leq n - 1$   
 $f(a_i u_{i+1}) = L_{3n+1+i}, 0 \leq i \leq n - 1$   
 $f(v_i w_i) = L_{4n+1+i}, 0 \leq i \leq n - 1$   
 $f(u_i w_i) = L_{5n+1+i}, 0 \leq i \leq n - 1$

The induced function  $f^*: V(G) \rightarrow \{1, 2, \dots, \sum L_q\}$  is given by  
 $f^*(v_i) = f(v_i b_{i+2}) + L_{i+1} + L_{4n+1+i}, 0 \leq i \leq n - 1$   
 $f^*(u_i) = f(u_i a_{i+2}) + L_{5n+1+i} + L_{2n+1+i}, 0 \leq i \leq n - 1$   
 $f^*(b_i) = L_{i+1} + L_{n+1+i}, 0 \leq i \leq n - 1$   
 $f^*(a_i) = L_{2n+1+i} + L_{3n+1+i}, 0 \leq i \leq n - 1$   
 $f^*(w_i) = L_{5n+1+i} + L_{4n+1+i}, 0 \leq i \leq n - 1$   
 We observe that the vertices are all distinct.  
 Hence  $G$  is a Lucas antimagic graph.

**Example 3.4:** The Lucas antimagic labeling of  $S(Gp(3,1))$  is shown below.

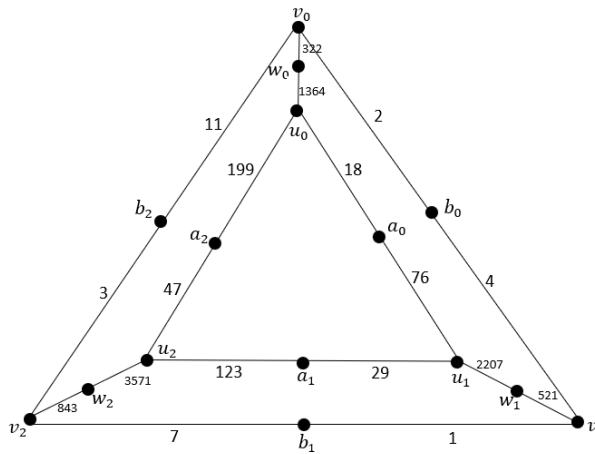


Figure 2 : Subdivision of Generalized Petersen graph  $S(Gp(3,1))$

**Theorem 3.5:**

The Complete bipartite graph  $K_{m,n}, m \geq 2, n \geq 2$  is a Lucas antimagic graph.

Let  $G$  be  $K_{m,n}$ .

Let  $V(G) = \{u_i : 1 \leq i \leq m, v_j : 1 \leq j \leq n\}$

$E(G) = \{u_i v_j : 1 \leq i \leq m, 1 \leq j \leq n\}$

Define a function  $f: E(G) \rightarrow \{L_1, L_2, \dots, L_q\}$  by

$f(u_1 v_1) = L_2$

$f(u_1 v_2) = L_1$

$f(u_1 v_j) = L_j, 3 \leq j \leq n$

$f(u_i v_j) = L_{(i-1)n+j}, 2 \leq i \leq m, 1 \leq j \leq n$

The induced function  $f^*: V(G) \rightarrow \{1, 2, \dots, \sum L_q\}$  is given by

$$f^*(u_1) = L_1 + L_2 + \sum_{j=3}^n L_j$$

$$f^*(u_i) = \sum_{j=1}^m L_{(i-1)n+j}, 2 \leq i \leq m$$

$$f^*(v_1) = L_2 + \sum_{i=2}^m L_{(i-1)n+1}$$

$$f^*(v_2) = L_1 + \sum_{i=2}^m L_{(i-1)n+2}$$

$$f^*(v_j) = \sum_{i=1}^m L_{(i-1)n+j}, 3 \leq j \leq n$$

We observe that the vertices are all distinct.  
Hence G is a Lucas antimagic graph.

**Example 3.6:** The Lucas antimagic labeling of Complete bipartite graph  $K_{3,3}$  is shown below.

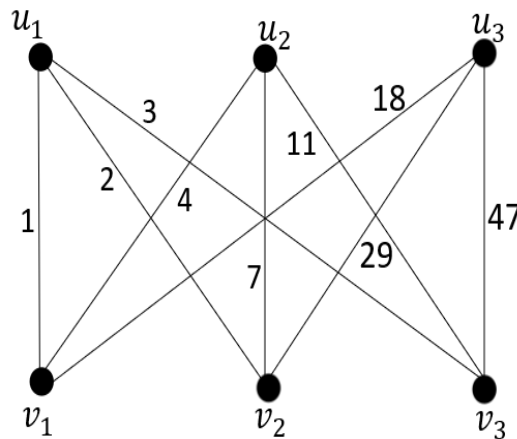


Figure 3: Complete bipartite graph  $K_{3,3}$

**Theorem 3.7:**

$S(K_{m,n}) (m \geq 2, n \geq 2)$  where  $S(G)$  denotes subdivision of  $G$  is a Lucas antimagic graph.

Proof:

Let G be  $S(K_{m,n})$

$$V(G) = \{u_i : 1 \leq i \leq m, v_j : 1 \leq j \leq n, v_j^i : 1 \leq i \leq m, 1 \leq j \leq n\}$$

$$E(G) = \{u_i v_j^i : 1 \leq i \leq m, 1 \leq j \leq n, v_j v_j^i : 1 \leq j \leq n, 1 \leq i \leq m\}$$

Define a function  $f: E(G) \rightarrow \{L_1, L_2, \dots, L_q\}$  by

$$f(u_i v_j^i) = L_{(i-1)n+j}, 1 \leq i \leq m, 1 \leq j \leq n$$

$$f(v_j v_j^i) = L_{mn+(j-1)n+i}, 1 \leq j \leq n, 1 \leq i \leq m$$

The induced function  $f^* : V(G) \rightarrow \{1, 2, \dots, \sum L_q\}$  is given by

$$f^*(u_i) = \sum_{j=1}^n L_{(i-1)n+j}, 1 \leq i \leq m$$

$$f^*(v_j) = \sum_{i=1}^m L_{mn+(j-1)n+i}, 1 \leq j \leq n$$

$$f^*(v_j^i) = L_{(i-1)n+j} + L_{mn+(j-1)n+i}, 1 \leq j \leq n, 1 \leq i \leq m$$

We observe that the vertices are all distinct.

Hence G is a Lucas antimagic graph.

**Example 3.8:** The Lucas antimagic labeling of Subdivision graph  $S(K_{2,2})$  is shown below.

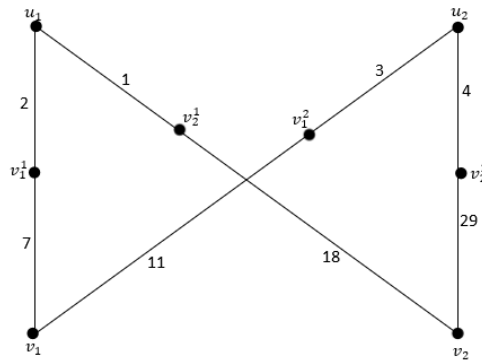


Figure 4: Subdivision of bipartite graph  $S(K_{2,2})$

#### IV. CONCLUSION

In this article, we establish that several generalized graphs, along with their subdivisions, exhibit Lucas Antimagic properties. Additionally, parallel investigations into various other graph structures are currently in progress, expanding the breadth of our research. These findings highlight the potential and applicability of Lucas Antimagic labeling in diverse graph theoretic contexts.

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