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Deriving Nu Equations from Convection and Conduction Equations

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ABSTRACT: This paper presents a derivation of the Nu equation directly from Newton's cooling law and Fourier's law, and the relation of the thicknesses of the velocity boundary layer and the thermal boundary layer. It is a more direct and logical approach, and the equations are more systematic. This approach is called "You's approach".

KEYWORDS: Nu equation, Newton's cooling law, Fourier's law, thicknesses of velocity boundary layer, and thicknesses of thermal boundary layer, You's approach

I. INTRODUCTION

In convection Heat transfer analysis, the primary problem is to determine the Heat transfer coefficient. There are five methods to determine the Heat transfer coefficient, and they are: dimensional analysis, exact solution of boundary layer equations, approximate solutions of boundary layer equations, Analogy between Momentum and heat transfer, and numerical analysis [1]. However, the dimensional analysis does not give an exact functional relation which can be solved. The exact solution is rather complex [1,2]. The accuracies of the approximate solutions depend on the assumptions of the of function of the velocity profile and the temperature profile [1]. Analogy between Momentum and heat transfer is a very useful tool for analysis of the turbulent flow process since we cannot write the equations completely for the turbulent flow yet, however, the analogy in the textbooks is from the equations already derived for Nusselt and for the friction coefficient. What we need is to derive the Nusselt equation from the friction coefficient. The numerical analysis is not presented in the standard heat transfer textbooks yet.

This paper presents a derivation of the Nu equation directly from Newton's cooling law and Fourier's law, and the relationship of the thicknesses of the velocity boundary layer and the thermal boundary layer. This approach makes the formulae of flow and heat transfer related more logically, and the science of heat transfer more systematic.

II. THE DERIVATION OF THE Nu EQUATIONS

2.1 The Derivation

The heat flux in convection heat transfer is (Newton's Law of Cooling):

$$\dot{q}_{conv} = h(T_{\infty} - T_s) \quad (1)$$

The heat transfer from the wall surface to the adjacent flow is by pure conduction, and the conduction heat flux at the wall surface is (Fourier's Law):

$$\dot{q}_{cond} = -k_f \frac{\partial T}{\partial y} \Big|_{y=0} \quad (2)$$

In steady state conditions, the heat transfer from the wall to the fluid by convection must be equal to the heat transfer conducted in the fluid layer, therefore, the heat transfer coefficient":

$$h = \frac{-k_f \frac{\partial T}{\partial y} \Big|_{y=0}}{(T_{\infty} - T_s)} = \frac{-k_f \frac{\partial(T - T_s)}{\partial y} \Big|_{y=0}}{(T_{\infty} - T_s)} = k_f \frac{\partial \left(\frac{T - T_s}{T_{\infty} - T_s} \right)}{\partial y} \Big|_{y=0} \quad (3)$$

The next task is to relate the heat transfer coefficient to the characteristics of the flow. From the definition of the thickness of the velocity boundary layer and the thermal boundary layer

$$\frac{\partial(\frac{T-T_s}{T_\infty-T_s})}{\partial y} \approx \frac{0.99}{\delta_t} \quad \text{and} \quad \frac{\partial(\frac{u}{u_\infty})}{\partial y} \approx \frac{0.99}{\delta}$$

Therefore,

$$\frac{\partial(\frac{T-T_s}{T_\infty-T_s})/\partial y}{\partial(\frac{u}{u_\infty})/\partial y} \approx \frac{\delta}{\delta_t} \quad (4)$$

From the solvation of the momentum equation and energy equation for the boundary layer on a flat plate [1,2], it is found that

$$\frac{\delta}{\delta_t} \approx Pr^{1/3} \quad (5)$$

Therefore,

$$h = k_f \frac{\partial(\frac{T-T_s}{T_\infty-T_s})}{\partial y} |_{y=0} = k_f \frac{\partial(\frac{u}{u_\infty})}{\partial y} |_{y=0} \frac{\delta}{\delta_t} = k_f \frac{\partial(\frac{u}{u_\infty})}{\partial y} Pr^{1/3} \quad (6)$$

or

$$Nu = \frac{hL}{k_f} = \frac{\partial(\frac{u}{u_\infty})L}{\partial y} Pr^{1/3} = \frac{(\frac{\partial u}{\partial y})L}{u_\infty} Pr^{1/3} = \frac{\mu(\frac{\partial u}{\partial y})L}{\mu u_\infty} Pr^{1/3} = \frac{\tau L}{\mu u_\infty} Pr^{1/3} \quad (7)$$

where $\tau = \mu \left(\frac{\partial u}{\partial y} \right)$ is the shear stress

2.2 For flat plate – Using Friction coefficient C_f

$$Nu = \frac{\tau L}{\mu u_\infty} Pr^{1/3} = \frac{C_f \rho u_\infty^2 L}{2 \mu u_\infty} Pr^{1/3} = \frac{C_f}{2} Re Pr^{1/3} \quad (8)$$

where $C_f = \frac{\tau}{\rho u_\infty^2}$ is the friction factor

For example:

For flat plate, laminar flow, the local friction coefficient [1,2]:

$$C_{fx} = 0.664 Re^{-1/2} \quad (9)$$

Therefore, the Local Nusselt Number

$$Nu_x = \frac{C_{fx}}{2} Re_x Pr^{1/3} = 0.332 Re_x^{1/2} Pr^{1/3} \quad (10)$$

Average friction coefficient

$$C_f = 1.328 Re^{-1/2} \quad (11)$$

Therefore, the average Nusselt Number

$$Nu = \frac{C_f}{2} Re Pr^{1/3} = 0.664 Re^{1/2} Pr^{1/3} \quad (12)$$

For flat plate, turbulent flow, the local friction coefficient [1,2]:

$$C_{fx} = 0.059 Re^{-1/5} \quad (13)$$

Therefore, the Local Nusselt Number

$$Nu_x = \frac{C_{fx}}{2} Re_x Pr^{1/3} = 0.0296 Re_x^{0.8} Pr^{1/3} \quad (14)$$

Average friction coefficient

$$C_f = 0.074 Re^{-1/5} \quad (15)$$

Therefore, the average Nusselt Number

$$Nu = \frac{C_f}{2} Re Pr^{1/3} = 0.037 Re^{0.8} Pr^{1/3} \quad (16)$$

This relationship can also be extended to the turbulent flow inside the tube.

2.3 For flows inside the tube – Using Friction factor f

For flow inside the tube, friction factor f is used to calculate the pressure drop:

$$\Delta P = f \frac{L \rho u_{\infty}^2}{D} \quad (17)$$

From the pressure balance relationship,

$$\Delta P \times \left(\frac{\pi D^2}{4}\right) = \tau \times (\pi DL) = \frac{C_f}{2} \rho u_{\infty}^2 \times (\pi DL) \quad (18)$$

i.e.:

$$f \frac{L \rho u_{\infty}^2}{D} \times \left(\frac{\pi D^2}{4}\right) = \frac{C_f}{2} \rho u_{\infty}^2 \times (\pi DL)$$

Therefore:

$$\frac{C_f}{2} = \frac{f}{8} \quad (19)$$

and

$$Nu = \frac{f}{8} Re Pr^{1/3} \quad (20)$$

For example:

For turbulent flow inside the tube

$$f = 0.184 Re^{-0.2} \quad (Re > 10,000) \quad (21)$$

Therefore,

$$Nu = \frac{f}{8} Re Pr^{1/3} = 0.023 Re^{0.8} Pr^{1/3} \quad (22)$$

This is known as Colburn equation.

The accuracy of Eq. 22 for turbulent flow is improved, and the more popularly used equation is the Dittus and Boelter equation:

$$Nu = 0.023 Re^{0.8} Pr^n \quad (23)$$

where $n=0.4$ for heating and $n=0.3$ for cooling of the fluid flowing through the pipe [1,2].

The error of Eq. 23 can also be furtherly improved by second Petukhov equation [1]:

$$Nu = \frac{f}{8} Re \frac{Pr}{1.07 + 12.7 \left(\frac{f}{8}\right)^{0.5} \frac{2}{(Pr^{2/3} - 1)}} \quad (24)$$

and

$$f = (0.79 \ln(Re) - 1.64)^{-2} \quad (3000 < Re < 5,000,000) \quad (25)$$

Calculation shows in the range of 3.85×10^4 to 9.52×10^5 , the value of f calculated from Eq. 25 is slightly smaller than that calculated from Eq. 21.

It can be seen that for the flows in the tube, either Pr^n in Eq. 23 or $\frac{Pr}{1.07 + 12.7 \left(\frac{f}{8}\right)^{0.5} \frac{2}{(Pr^{2/3} - 1)}}$ in Eq.24, is the correction to $Pr^{1/3}$ in Eq.22.

2.4 j -factor for heat transfer (j_H) and its relationship to Nu

It is defined that [1,2]:

$$St = \frac{Nu}{Re Pr} \quad (26)$$

Therefore,

$$Nu = St Re Pr \quad (27)$$

From Eq. 26 and Eq. 8,

$$Nu = St Re Pr = \frac{C_f}{2} Re Pr^{1/3} \quad (28)$$

It can be seen that

$$\frac{C_f}{2} = St Pr^{2/3} \quad (29)$$

and

$$\frac{f}{8} = StPr^{2/3} \tag{30}$$

j_H -factor for heat transfer (or Colburn factor) j_H

$$j_H = StPr^{2/3} = \frac{C_f}{2} = \frac{f}{8} \tag{31}$$

$$\therefore Nu = j_H Re Pr^{1/3} \tag{32}$$

Therefore, for a new convention heat transfer problem, it is important to find out its j_H . The equations of the friction coefficient (factor) and the Nu equations are summarized in the table below.

Table 1. Summary of the equations of the friction coefficient (factor) and the Nu equations

Flow Over Flat plates	Friction Coefficient	Nusselt Number $Nu = \frac{C_f}{2} Re Pr^{1/3}$
Laminar flow, Local	$C_{fx} = 0.664 Re^{-1/2}$	$Nu_x = 0.332 Re_x^{1/2} Pr^{1/3}$
Laminar flow, Average	$C_f = 1.328 Re^{-1/2}$	$Nu = 0.664 Re^{1/2} Pr^{1/3}$
Turbulent flow, Local	$C_{fx} = 0.059 Re^{-1/5}$	$Nu_x = 0.0296 Re_x^{0.8} Pr^{1/3}$
Turbulent flow, Average	$C_f = 0.074 Re^{-1/5}$	$Nu = 0.037 Re^{0.8} Pr^{1/3}$
Flow inside tubes	Friction Factor	Nusselt Number $Nu = \frac{f}{8} Re Pr^{1/3}$
Turbulent flow, simple one	$f = 0.184 Re^{-0.2}$	$Nu = 0.023 Re^{0.8} Pr^n$
Turbulent flow, more accurate one	$f = (0.79 \ln(Re) - 1.64)^{-2}$	$Nu = \frac{f}{8} Re \frac{Pr}{1.07 + 12.7 \left(\frac{f}{8}\right)^{0.5} (Pr^{2/3} - 1)}$

III. CONCLUSION

This paper presents a derivation of the Nu equation directly from Newton’s cooling law and Fourier’s law, and the relationship of the thicknesses of the velocity boundary layer and the thermal boundary layer. This approach makes the formulae of flow and heat transfer related more logically, and the science of heat transfer more systematic. This approach is called “You’s approach”.

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