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## Self Focusing of Laser Beam in Linearly Increasing and Decreasing Inhomogeneity in Plasma using Non-Paraxial Method

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**ABSTRACT:** One of the most significant factors in understanding the behaviour of laser beams during the propagation process through plasma is how the spot size of the beam changes with both laser and plasma parameters. Self-focusing and defocusing of the laser beam, therefore, play an important role in this study. This chapter is aimed at presenting a brief investigation into these common phenomena in laser-plasma interaction. In addition to a short overview on the types of self-focusing of laser beams through plasma, a detailed study on the relativistic self-focusing of high-intense laser beam in quantum plasma as a particular nonlinear medium is performed. In this case, the essential equations to model this phenomenon are derived; furthermore, a range of values of laser-plasma parameters which would satisfy these equations is considered.

KEYWORDS: laser beams, plasma, non-paraxial, inhomogeneity, self-focusing

## I. INTRODUCTION

Wave propagation in optical fiber has been a subject of intensive theoritical and experimental research [1-10]. Optical fiber technology changed the communication scenario and efforts are on, in achieving the goal of dispersionless transmission over a long distance [11-14]. Focusing due to self-action has been employed for dispersionless propagation of laser beam in non-linear media [15–17]. Trapping of beam in dielectric cell had been demonstrated experimentally by Chiao et al [18] and Gramire et al [19]. Propagation of intense laser beam and its trapping in optical fibers due to self-action is of current interest because of its potential applications in optical switching, Kerr lens mode locking, optical power filter, fiber amplifiers etc. [20]. An intense laser beam with radial distribution of intensity causes a radial gradient of dielectric constant due to non-linear mechanisms such as Kerr effect, electrostriction, thermal effects etc. This leads to focusing or defocusing of beam depending on the nature of the non-linear medium in which the beam is propagating. Theories of self-focusing (focusing of beam due to self-action) have relied on the paraxial ray approximation [21–24]. These approximations are known to give large error in the critical power for self-focusing [19]. Realising that paraxial approximation may also be quantitatively in error in the saturation region, some alternative methods for self-focusing and self-trapping of beam in plasma had been suggested and used by various workers [25– 30]. The self-focusing problem of non-linear interaction of intense laser beam has been analysed considering the entire spatial characteristics of the laser beam, without any paraxial ray approximation and Taylor series expansion of dielectric constant. The effect of the Kerr non-linearity on fiber parameter and propagation characteristics of laser beam have been discussed. Results of the present analysis are compared with popular paraxial ray theories [24]. The paper is organized as follows: After the introduction in 1, the mathematical expression for effective dielectric constant has been presented in 2. Section 3 is devoted to the self-focusing equations considering non-saturating Kerr non-linearity. Second-order differential equations related to step-index as well as parabolic profile fiber have been obtained and presented in this section. In 4 the condition for uniform waveguide propagation of laser beam in optical fiber is established and expression for critical power is also obtained. Numerical results of self-action effects are presented in 5. Details of results and related conclusions are presented in 6. The non-stationary self-focusing of high intense laser beam in under-dense plasma with upward increasing density ramp is investigated. The obtained results show that slowly increasing plasma density ramp is very important in enhancing laser self-focusing. Also, the spot size oscillations of laser beam in front and rear of the pulse for two different density profiles are shown. We have selected density profiles that already were used by Sadighi-Bonabi et al. [Phys. Plasmas 16, 083105 (2009)]. Ramp density profile causes the laser beam to become more focused and penetrations deeps into the plasma by reduction of diffraction effects. Our computations show more reliable results in comparison to the previous works. The interaction





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of intense laser beams with plasmas is important for a number of applications such as particle acceleration by plasma waves [1], [2], [3], [4] for inertial confinement fusion. These applications require the beam to propagate over several Rayleigh lengths without loss of energy. When a high power laser beam is involved, then it can cause an electron oscillatory velocity comparable to the velocity of light, which modifies the effective dielectric constant of the plasma; this mechanism is responsible for relativistic self-focusing of laser beam [5]. Relativistic laser-plasma interaction has been studied in detail by many authors, both theoretically [5], [6], [7] and, experimentally [8], [9], [10] and reviewed elegantly [11], [12]. Pukhov and Meyer-ter-vehn [13] reported three dimensional particle in cell (3D PIC) simulations for laser propagation in a near critical underdense plasma far above the threshold for relativistic filamentation. The key feature predicted by these simulations is the formation of the narrow, single propagation channel, containing the significant part of laser energy. The simulation reveals the importance of relativistic electrons traveling with light pulse and generating multimega gauss magnetic field that strongly influences the light propagation. In order to generate relativistic electron oscillation by laser beams in plasmas, dielectric increase and hence self-focusing are desirable [5]. A review of literature highlights the fact that the self-focusing of laser beams in plasmas is confined to the classical relativistic regime. Quantum plasmas are encountered in many environments, i.e., in ultrasmall electronic devices such as carbon nanotubes [14], quantum dots [15] and biophotonics [16], and also in dense astrophysical bodies and in lasersolid interactions [17]. Quantum plasmas are characterized by low-temperature and high density and can be studied via the Wigner formalism, which is the quantum equivalent to the classical Vlasov equation [18]. Quantum effects can be measured by the thermal de Broglie wavelength of the particles in the plasma  $\lambda B = \hbar/mvf$ , where vf=(kBTf/m)1/2 is the Fermi thermal velocity and  $T_f$  is the Fermi temperature. According to statistical mechanics, quantum effects become important when the temperature of the system is lower than T<sub>f</sub>. In classical regimes, the de Broglie wavelength is so small that particles can be considered point-like and thus no quantum interference occurs. On the other hand, quantum effects enter into play when the de Broglie wavelength becomes of the order of (or larger than) the average interparticle distance [18]. Within the quantum hydrodynamical description, quantum effects are elegantly modeled by the Bohm potential [19], $\phi B = -\hbar 22$  mme $\nabla 2$  newhich modifies the linear wave characteristics [20]. Here n<sub>e</sub> is the density of plasma electrons and m is the mass of electron.

In the present paper, authors have studied evolution of Gaussian laser beam in relativistic cold quantum plasma through parabolic equation approach. In common with most investigations the present analysis makes use of the paraxial approximation.[1,2,3]

## **II. DISCUSSION**

There are a lot of non-linear phenomena in the interaction between intense laser and plasma, such as self-focusing/selfdefocusing [Reference Kovalev and Bychenkov1], filamentation [Reference Hassan, Abd-Ali and Soary2], stimulated Raman scattering [Reference Peng, Zuo, Zhu and Su3] and stimulated Brillouin scattering [Reference Li, Zuo, Su and Yang4]. These nonlinearities have important effects on inertial confinement fusion [Reference Lu, Tikhonchuk and Weber5] and laser particle accelerator [Reference Lotov, Sosedkin and Petrenko6]. In particular, plasma self-focusing can improve the yield of these applications by increasing the interaction length between the laser and the medium, but also increase the success of these applications by balancing the natural diffraction of the laser beam to transmit more Rayleigh lengths [Reference Asgharnejad, Mohsenpour and Mirzanejhad7]. Therefore, the self-focusing generated by the interaction of laser pulses with plasma is a very worthy topic of study [Reference Xia and Xu8].

In interaction intense laser and plasms, the mechanism of self-focusing/defocusing mainly include collisional nonlinearity [Reference Singh and Gupta9], ponderomotive nonlinearity [Reference Patil, Chikode and Takale10, Reference Rawat and Purohit11], relativistic nonlinearity [Reference Sharma and Tripathi12]. Nanda and Kant [Reference Nanda and Kant13] researched self-focusing of cosh-Gaussian laser beams in collisionless non-uniform magnetic plasmas. By comparing extraordinary mode and ordinary mode, it is found that increasing the plasma density ramp can make the former stronger self-focusing. Asgharnejad et al. [Reference Asgharnejad, Mohsenpour and Mirzanejhad7] investigated self-focusing a Gaussian laser pulse in a weakly relativistic and ponderomotive regime inside a collisionless warm quantum plasma. Zhang et al. [Reference Zhang, Liang and Xia14] analyzed relativistic self-focusing of Gaussian laser beam, the results show that relativistic effect can enhance the oscillation of dielectric function and increase the self-focusing. Abedi-Varaki and Jafari [Reference Abedi-Varaki and Jafari15] have present that a decrease in plasma temperature and an increase in oblique magnetic field increases defocusing for left-handed polarization. Extending the High-harmonic generation phase matching cutoff by controlling the rapid self-defocusing





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effect of driving laser is experimentally and theoretically demonstrated by Sun et al. [Reference Sun, Huang and Tzeng16].

Lots of research of self-focusing/self-defocusing have focused on interaction of Gaussian beam and homogeneous/inhomogeneous plasmas. Studies by Gupta et al. [Reference Gupta, Hur and Suk17] and Kant et al. [Reference Kant, Wani and Kumar18] have shown that the existence of plasma density fluctuations can enhance selffocusing. Wani et al. [Reference Wani, Thakur, Ghotra and Kant19] have found that electron temperature can reduce defocusing and plasma density transition increases the range of self-focusing effect. Recently, many researchers have analyzed the effects of different types of laser beams and plasma on self-focusing/self-defocusing. Laser beam extends from Gaussian type to the others type of beam, such as Hermite-Gaussian beam [Reference Wadhwa and Singh20], cosh-Gaussian beam [Reference Urunkar, Patil, Valkunde, Vhanmore, Gavade and Takale21], super Gaussian beam [Reference Malik and Devi22]. Various types plasmas have been researched, such as cold quantum plasma with exponential density variation [Reference Thakur and Kant23]. Effects of magnetic field on relativistic self-focusing, self-phase modulation and self-trapping of cosh-Gaussian laser beam is exploited by Gill et al. [Reference Gill, Kaur and Mahajan24]. Kant et al. [Reference Kant, Vij, Chakravarti, Kushwaha and Thakur25] [4,5,6]observed relativistic self-focusing of Hermite-cosh-Gaussian laser pulse in plasma with exponential plasma density ramp. Devi and Malik [Reference Devi and Malik26] studied the relativistic self-focusing of super-Gaussian laser beams, they have found that the external magnetic field and the cyclotron motion of electrons is important for self-focusing. Based on the moment theory, Malik and Devi [Reference Malik and Devi27] developed a formulation to study the self-defocusing of super-Gaussian laser beam in tunnel ionized plasmas, and found that laser beam can get more defocused for higher intensity and smaller spot size. Kaur et al. [Reference Kaur, Singh Gill and Mahajan28] used paraxial approximation method to study propagation characteristics of Hermite-cosh-Gaussian laser beam in rippled density plasmas and strong oscillatory self-focusing and defocusing are observed in relativistic case.

In this paper, applying higher-order paraxial theory and WKB approximation, the self-focusing/self-defocusing are investigated in the interaction of Hermite-Sinh-Gaussian (HshG) laser beam and the non-uniform underdense plasma with periodic density fluctuations.

2. Propagation Theory of Laser Beam in Plasmas

In laser-plasma interaction, according to Maxwell's equation theory, the wave equation of laser propagation in plasma can be expressed as,

$$\nabla^{2}E - \frac{\omega^{2}}{c^{2}}\varepsilon E + \nabla\left(\frac{E\cdot\nabla(\varepsilon)}{\varepsilon}\right) = 0.$$

For  $k^{-2}\nabla^2 \ln(\varepsilon) \ll 1_{\text{and}} k^2 = \varepsilon \omega^2/c^2$ , the wave equation can be converted to

$$\frac{\partial^2 E}{\partial z^2} + \frac{\partial^2 E}{\partial r^2} + \frac{1}{r} \frac{\partial E}{\partial r} + \frac{\omega^2}{c^2} \varepsilon E = 0.$$

When the laser propagates in the plasma along the z-direction, the specific form of the field can be expressed as,

$$_{(3)}E = A(r,z)e^{-i(\omega t - kz)}.$$

In this paper, the interaction between HshG laser beam and inhomogeneous plasma is studied. For equation (3), the general solution of the field amplitude is HshG beam.

$$E(r,z) = \frac{E_0}{f(z)} \left[ H_n \left( \frac{\sqrt{2} r}{r_0 f(z)} \right) \right] e^{b^2/4} \left( e^{-\left( \left( r/r_0 f \right) + \left( b/2 \right) \right)^2} - e^{-\left( \left( r/r_0 f \right) - \left( b/2 \right) \right)^2} \right).$$

(2)





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E<sub>0</sub> is the amplitude of HshG laser beam at r = z = 0, Hermite polynomial  $H_n(\sqrt{2}r/r_0f(z))$ , when n = 0,  $H_n(\sqrt{2}r/r_0f(z)) = 1$ ; when n = 1,  $H_n(\sqrt{2}r/r_0f(z)) = 2r$ ; when n = 1 $H_n(\sqrt{2}r/r_0f(z)) = 4r^2 - 2$ , r is the radial component in the cylindrical coordinate system, r<sub>0</sub> is the initial beam width, b represents decentred parameter, the dimensionless beam width parameter of laser beam is f(z). Non-uniform plasmas showing periodic variations have been found experimentally and theoretically by researchers in

the process of laser-plasma interaction. The plasma system studied in this paper is based on the experimental results of Ouahid et al. [Reference Ouahid, Dalil-Essakali and Belafhal29]; electron density  $N_e(z)$  is sinusoidal function. The expression is as follows,

## $_{(5)}N_e(z) = N_{0e}(1 + \alpha \sin qz),$

where Ne(z), N 0e, a, q represents electron density in equilibrium steady state, initial electron density, parameter of plasma fluctuation, wave number of plasma density fluctuation.[7,8,9]

In laser-plasma interaction, when plasma density fluctuation exists, the dielectric function of plasma composed of linear part  $\varepsilon_0$  and non-linear part  $\phi(EE^*)$  can be expressed as  $\varepsilon(r, z) = \varepsilon_0 - \phi(EE^*)$ . In the limit  $c^2/(r_0^2\omega_{p0}^2) < 1$ limit s

follows

, the linear and non-linear part can be presented a
$$\varepsilon_0(z) = 1 - \omega_{p0}^2 (1 + \alpha \sin qz) / \gamma \omega^2$$

 $\phi(EE^*) = \omega_{p0}^2 (1 + \alpha \sin qz) [1 - 1/(1 + \beta EE^*)^{1/2}] / \gamma \omega^2 \quad \omega_{p0} = \sqrt{4\pi n_0 e^2/m}$  is the plasma frequency, e is the electronic charge and m shows the rest mass. Here the relativistic factor can be written

as  $\gamma = \sqrt{1 + \beta E E^*/2}$ , where  $\beta = (e/m\omega c)^2$ . By applying higher-order paraxial theory and extending the dielectric function up to the fourth higher-order term.

$$\varepsilon(r, z) = \varepsilon_0(z) - \varepsilon_2(z) \frac{r^2}{r_0^2} - \varepsilon_4(z) \frac{r^4}{r_0^4}$$

where  $\varepsilon_0(z)$  is the linear part,  $\varepsilon_2(z)$  and  $\varepsilon_4(z)$  are expansion coefficients of  $\varepsilon(r, z)$ . Based on the research results of Pathak et al. [Reference Pathak, Kaur, Kaur and Gill30]; using WKB approximation and higher-order paraxial theory, dielectric constant components of different Hermite mode indexes can be obtained in interaction of HshG beams and plasma. For mode index n = 0

$$\begin{cases} \varepsilon_{0} = 1 - t + \frac{3t}{4} \frac{B}{f^{2}} e^{b^{2}/2} D - \frac{27t}{32} \frac{B^{2}}{f^{4}} e^{b^{2}} D^{2}, \\ \varepsilon_{2} = t \frac{B^{2}}{f^{6}} e^{b^{2}} \Big[ \frac{3}{32} (4b^{2} - 36) D + \frac{27}{16} a_{2} D^{2} \Big] - \frac{tB}{4f^{4}} e^{b^{2}/2} \Big[ (7b^{2} - 6) + 3a_{2} D \Big], \\ \varepsilon_{4} = \frac{tB^{2}}{f^{8}} e^{b^{2}} \Big[ \frac{3}{32} (72 + \frac{107}{2} b^{4} - 102b^{2}) + \frac{27D^{2}}{32} (a_{2}^{2} + 2a_{4}) + \frac{3}{16} a_{2} (42b^{2} - 36) \Big] \\ - \frac{tB}{4f^{6}} e^{b^{2}/2} \Big[ 6 + a_{2} (7b^{2} - 6) + 3a_{4} D \Big]. \end{cases}$$

For mode index n = 1

(7)





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$$\begin{cases} \varepsilon_{0} = 1 - t, \\ \varepsilon_{2} = -\frac{6tB}{f^{4}}e^{b^{2}/2}D, \\ \varepsilon_{4} = \frac{54tB^{2}}{f^{8}}e^{b^{2}}D^{2} - \frac{2tB}{f^{6}}e^{b^{2}/2}[(7b^{2} - 6) + 3a_{2}D]. \end{cases}$$
(8)

For mode index n = 2 (9)

$$\begin{split} & \left\{ \begin{aligned} \varepsilon_{0} &= 1 - t + \frac{3tB}{f^{2}} e^{b^{2}/2} D - \frac{27t}{2} \frac{B^{2}}{f^{4}} e^{b^{2}} D^{2}, \\ & \varepsilon_{2} &= \frac{9t}{4} \frac{B^{2}}{f^{6}} e^{b^{2}} \left[ 4 \left( 7b^{2} - 6 \right) - 96 D + 12a_{2} D \right] - \frac{tB}{4f^{4}} e^{b^{2}/2} \left[ 4 \left( 7b^{2} - 6 \right) - 96 D + 12a_{2} D \right], \\ & \varepsilon_{4} &= \frac{3tB^{2}}{32f^{8}} e^{b^{2}} \left[ 4 \left( 7b^{2} - 6 \right) - 96 D + 12a_{2} D \right]^{2} + \frac{9tB^{2}D}{4f^{8}} e^{b^{2}} \left[ 24 + (192 - 96a_{2} + 12a_{4}) D + (4a_{2} - 32) \left( 7b^{2} - 6 \right) \right], \\ & - \frac{tB}{4f^{6}} e^{b^{2}/2} \left[ 24 + (192 - 96a_{2} + 12a_{4}) D + (4a_{2} - 32) \left( 7b^{2} - 6 \right) \right], \end{split}$$

where in equations (7)–(9),  $t = \omega_{p0}^2/\omega^2 (1 + \alpha \sin(q\xi))$  denotes the parameters related to plasma density,  $B = \beta E_{0}^2$ ,  $D = 1 - b^2/2 + b^4/8$  denote intensity dependence and intensity distribution dependence of dielectric constant. For wave equation (2), substitute (3) into equation (2),  $\partial^2 A/\partial z^2$  be ignored, can get,

$$-2ik\frac{\partial A}{\partial z} - iA\frac{\partial k}{\partial z} + \left(\frac{\partial^2 A}{\partial r^2} + \frac{1}{r}\frac{\partial A}{\partial r}\right) + \frac{\omega^2}{c^2} \left[\varepsilon(r,z) - \varepsilon_0(z)\right]A = 0.$$

Following Nanda et al. [Reference Nanda, Ghotra and Kant31]; complex amplitude A can be written as follows,

 $A(r,z) = A_0(r,z)e^{-ikS(r,z)}$ ,  $A_0(r,z)$  and S(r,z) are the real functions of space, according to Habibi and Ghamari [Reference Habibi and Ghamari32]; they can be expressed this way[10,11,12]





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$$A_{0}^{(12)} = \frac{E_{0}^{2}}{f^{2}} e^{b^{2}/2} \left( H_{n} \left( \frac{\sqrt{2} r}{r_{0} f} \right) \right)^{2} \left( 1 + \frac{a_{2} r^{2}}{r_{0}^{2} f^{2}} + \frac{a_{4} r^{4}}{r_{0}^{4} f^{4}} \right) \left[ e^{-2 \left( \left( r/r_{0} f \right) + \left( b^{2}/2 \right) \right)} - e^{-\left( \left( 2r^{2}/r_{0}^{2} f^{2} \right) + \left( b^{2}/2 \right) \right)} \right],$$

$$S(r, z) = S_{0}(z) + \frac{r^{2}}{r_{0}^{2}} S_{2}(z) + \frac{r^{4}}{r_{0}^{4}} S_{4}(z).$$

$$(13) \qquad \text{a.s. a.t. } S_{0}, S_{2}, S_{4} \text{ are } \text{the functions } of z, S_{0}, S_{2}, S_{4} \text{ are } \text{the components } of \text{ the }$$

a 2, a 4, S 0, S 2, S 4 are the functions of z, S 0, S 2, S 4 are the components of the spherical curvature of the wave front, S 4 indicates deviation  $S_2 = (r_0^2/2f)(df/d\xi)$  corresponds to the spherical curvature of the wave front, S 4 indicates deviation

eikonal. corresponds to the spherical curvature of the wave front, S  $_4$  indicates deviation from spherical curvature. a  $_2$ , a  $_4$  are indicative of the departure of the beam from the HshG nature. Can get a complicated differential equation by bringing in equation (10) with equation (11), the following equations can be obtained if the real and imaginary parts are equal.

$${}_{(14)} 2\frac{\partial S}{\partial z} + \frac{2S}{k}\frac{\partial k}{\partial z} + \left(\frac{\partial S}{\partial r}\right)^2 = \frac{1}{k^2 A_0} \left(\frac{\partial^2 A_0}{\partial r^2} + \frac{1}{r}\frac{\partial A_0}{\partial r}\right) - \frac{r^2}{r_0^2}\frac{\varepsilon_2(z)}{\varepsilon_0(z)} - \frac{r^4}{r_0^4}\frac{\varepsilon_4(z)}{\varepsilon_0(z)},$$
$$\frac{\partial A_0^2}{\partial z} + A_0^2 \left(\frac{\partial^2 S}{\partial r^2} + \frac{1}{r}\frac{\partial S}{\partial r}\right) + \frac{\partial A_0^2}{\partial r}\frac{\partial S}{\partial r} + \frac{A_0^2}{k}\frac{dk}{dz} = 0.$$

By substituting equations (12) and (13) in equation (15) for different Hermite mode indexes, and equating the coefficients of  $r^2$  and  $r^4$  on both sides of equation, the equation relation of parameter  $a_2$  and  $a_4$  is represented as.

For mode index n = 0  

$$\frac{da_2}{d\xi} = -16S'_4 f^2,$$

$$\frac{da_4}{d\xi} = \left[-24a_2 + 4\left(4 - 2b^2\right)\right]S'_4 f^2,$$

For mode index n = 1

$$\frac{da_2}{d\xi} = -24S'_4f^2,$$
  
$$\frac{da_4}{d\xi} = \left[-32a_2 + 4\left(4 - 2b^2\right)\right]S'_4f^2.$$

For mode index n = 2

$$\frac{da_2}{d\xi} = -16S'_4 f^2,$$
  
$$\frac{da_4}{d\xi} = \left[-24a_2 + 64 + 4\left(4 - 2b^2\right)\right]S'_4 f^2,$$

where  $S'_4 = S_4(\omega/c)$ . Same method, equations (12) and (13) in equation (14) to get the following equations governing the beam width f and  $S'_4$ . For mode index n = 0





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$$\frac{d^2f}{d\xi^2} = \left(\frac{-2a_2^2 + 8 - 8a_2 - 46b^4/3 + 4a_2b^2 - 8b^2 + 8a_4}{\varepsilon_0 f^3}\right) - \frac{\varepsilon_2 f\rho^2}{\varepsilon_0} - \frac{1}{2\varepsilon_0} \frac{d\varepsilon_0}{d\xi} \frac{df}{d\xi},$$
$$\frac{dS_4'}{d\xi} = \frac{8a_2a_4 - 4a_2^3 - 52a_2b^4/3 - 16a_4 + 8a_2^2 + 104b^4/3 + 8b^2a_4 - 4a_2^2b^2 - 52b^6/3}{2\varepsilon_0 f^6}$$

$$-\frac{4S_4'}{f}\frac{df}{d\xi}-\frac{S_4'}{2\varepsilon_0}\frac{d\varepsilon_0}{d\xi}-\frac{\varepsilon_4\rho^2}{2\varepsilon_0}.$$

For mode index n = 1

$$\frac{d^{2}f}{d\xi^{2}} = \left(\frac{8 - 6a_{2}^{2} - 8a_{2} - 98b^{4}/3 + 4a_{2}b^{2} - 8b^{2} + 16a_{4}}{\varepsilon_{0}f^{3}}\right) - \frac{\varepsilon_{2}f\rho^{2}}{\varepsilon_{0}} - \frac{1}{2\varepsilon_{0}}\frac{d\varepsilon_{0}}{d\xi}\frac{df}{d\xi},$$
$$\frac{dS_{4}'}{d\xi} = \frac{(2a_{2} - 4 + 2b^{2})(4a_{4} - 2a_{2}^{2} - 26b^{4}/3)}{2\varepsilon_{0}f^{6}} - \frac{4S_{4}'}{f}\frac{df}{d\xi} - \frac{S_{4}'}{2\varepsilon_{0}}\frac{d\varepsilon_{0}}{d\xi} - \frac{\varepsilon_{4}\rho^{2}}{2\varepsilon_{0}}.$$

For mode index n = 2  

$$\frac{d^{2}f}{d\xi^{2}} = \left(\frac{-2a_{2}^{2} + 72 - 40a_{2} - 46b^{4}/3 + 4a_{2}b^{2} - 40b^{2} + 8a_{4}}{\varepsilon_{0}f^{3}}\right) - \frac{\varepsilon_{2}f\rho^{2}}{\varepsilon_{0}} - \frac{1}{2\varepsilon_{0}}\frac{d\varepsilon_{0}}{d\xi}\frac{df}{d\xi},$$

$$\frac{dS_{4}'}{d\xi} = \frac{(2a_{2} - 20 + 2b^{2})(4a_{4} - 64 - 2a_{2}^{2} - 26b^{4}/3)}{2\varepsilon_{0}f^{6}} - \frac{4S_{4}'}{f}\frac{df}{d\xi} - \frac{S_{4}'}{2\varepsilon_{0}}\frac{d\varepsilon_{0}}{d\xi} - \frac{\varepsilon_{4}\rho^{2}}{2\varepsilon_{0}}.$$

Normalize the above equations,  $\xi = cz/\omega r_0^2 = z/R_d$  is the dimensionless propagation distance,  $\rho = \omega r_0/c$  represents the dimensionless original beam width.[13,14,15]

#### **III. RESULTS**

In the paper, intensity of incident laser beam is  $1.23 \times 10^{17}$  W/cm<sup>2</sup>,  $\beta E_0^2 = 0.01, 0.02, 0.1$  and 0.2 (corresponding intensities are  $1.23 \times 10^{15}$  W/cm<sup>2</sup>,  $2.46 \times 10^{15}$  W/cm<sup>2</sup>,  $1.23 \times 10^{16}$  W/cm<sup>2</sup> and  $2.46 \times 10^{16}$  W/cm<sup>2</sup> respectively), the frequency and spot size of incident laser beam as  $\omega = 1.78 \times 10^{15}$  rad/s,  $r_0 = 1.68 \times 10^{-5}$  m in interaction system. Appling fourth-order Runge-Kutta method to solve equations (16a)–(21b), the phenomena of self-focusing/self-defocusing in the interaction of intense laser and the inhomogeneous plasma are discussed. The initial conditions are f = 1,  $df/d\xi = 0$ ,  $a_2 = 1$ , as well as  $da_2/d\xi = 0$  at  $\xi = 0$ , plasmas density is  $N_e(z) = N_{0e}(1 + \alpha \sin qz)$ .

Figures 1(a)–1(c) shows the linear component of the dielectric function of the HshG laser beam interaction of plasma with periodic density fluctuation at mode index n = 0, 1 and 2. The linear part of dielectric function shows sinusoidal periodic variations similar to oscillation in plasma of periodic density fluctuation. The peak value of the linear part decreases slightly with the increase of mode index, and the oscillation has intensified the tendency. For the non-linear part  $\varepsilon_2$ ,  $\varepsilon_4$  at different mode indexes, the value is much smaller compared to the linear part, and mode index has a more obvious influence on the non-linear part. In figure (d), (e) and (f), it can be seen that mode index obvious affect the peak value of non-linear parts of the dielectric function, it shows an approximate periodic oscillatory variation due to the plasma periodicity, and its oscillatory variation trend to increase with the increase of mode index of





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Hermite polynomial. In addition, the mode index have more obvious effect on the non-linear part comparison linear part. In the periodic underdense inhomogeneous plasma, the beam width all show oscillatory changes, firstly, beam width becomes small and then increases. During the propagation of the laser beam, ponderomotive force influence on laser beam is obvious at the start, and beam width reduces and appears to convergence, resulting in self-focusing. Then, the influence ponderomotive force of laser beam decrease and beam width increase continuously with distance, and beam width presents diverge obviously, resulting in beam self-defocusing. However, at different mode indexes, the size and position of the minimum value of beam width are different. Increase of mode index, the size slightly increase and position appear early, and the beam diverges more quickly, which means the self-defocusing is more intense. For n = 0, reaches the minimum value 0.55 at  $\xi \approx 0.14$ . For n = 1, reaches the minimum value 0.6 at  $\xi \approx 0.11$ . For n = 2, reaches the minimum value 0.998 at  $\xi \approx 0.02$ . This is due to the combined effects of HshG laser beam and periodic underdense non-uniform plasma, causing a similar periodic oscillatory variation of dielectric function, which leads to self-focusing and self-defocusing. In addition with the increase of mode index the oscillation of the dielectric function.

underdense non-uniform plasma, causing a similar periodic oscillatory variation of dielectric function, which leads to self-focusing and self-defocusing. In addition, with the increase of mode index, the oscillation of the dielectric function is enhanced, indicating that the instability is enhanced, which will affect the coupling of the laser plasma, so that the quality of Gaussian beam decreases, self-focusing reduce and self-defocusing enhance.[16,17,18]

## **IV. CONCLUSION**

In the paper, apply high-order paraxial theory to analyze the self-focusing/self-defocusing of HshG laser beam in underdense inhomogeneous plasma. Self-focusing and self-defocusing laws are explored by analyzing the variation of the dielectric constant and the influence of the decentered parameters, laser intensity and plasma density on beam width. The results show that, due to the combined effects of HshG laser beam and periodic underdense non-uniform plasma, lead to dielectric function present a similar periodic oscillation variation. Under the influence of ponderomotive force and diffraction effect, lead to beam self-focusing/self-defocusing. [19]With increase of Hermite mode index, causing the position of self-focusing/self-defocusing is bring forward, and beam defocusing is strengthened, it implies that the higher mode index HshG beam is benefit to reduce self-focusing and enhance defocusing. In addition, the decentered parameter, beam intensity and non-uniformity of the plasma has a significant influence on the beam width. Influence of the decentered parameter on beam self-focusing is stronger than self-defocusing. In the high-order HshG beam, high beam intensity is benefit to restrain self-focusing and enhance self-defocusing, and increase non-uniformity of plasmas, it can strength self-focusing/self-defocusing.[20]

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