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ijirset@gmail.com

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Sequential Game and Nash Equilibrium: their Applications

Sri Naren Chandra Boro

Assistant Professor, Department of Mathematics, Rabindranath Tagore University, Hojai, Assam, India

ABSTRACT: Game theory is the branch of mathematics that studies the strategies involved in decision making. In this paper I will highlight the basic concept, modeling tools of game theory and various game strategies for solving a game. Here I will basically highlight the sequential game and Nash equilibrium and few applications of them in various fields. Game theory is a useful concept in modern science that is useful in fields such as psychology, biology, political science, computer science, sociology, economics, and diplomacy and milliliter strategy. It is used to find optimal outcome from a set of choices by analyzing the cost and benefits to such independent competition.

KEY WORDS: Game, strategy, dominated strategy, sequential move, Nash equilibrium.

I. INTRODUCTION

In our day to day life we always make decisions that can have immediate or long term effects in our lives. Game theory is a branch of mathematics that helps in studying the strategic decision making in various situations of conflict. A game is an abstract of strategic situations involving inter dependence. A simple form of game is defined by players, strategies, payoffs etc. A game makes explicit that rules governing players interactions, the players feasible strategies and their performances over outcome. Game theory provides a new perspective and method for solving various decision making problem. In game theory one important concept is the Nash Equilibrium that has been using widely in various fields such as economics, political science, sociology, psychology etc. In economics, it has been widely applied to analyze market competition, pricing strategies, industrial organizations, etc. Here in this paper I will highlight the game strategies with special example of Prisoner's dilemma and sequential move in a game and their effects in Nash equilibrium. At last few examples as its applications in the field of social sciences. I hope the study of this paper researchers will gain a different idea in understanding the game, various strategies of game and how they are useful in problem solving in various fields.

II. BASIC TERMS ASSOCIATED WITH GAME

Player: The competitors in the game are known as players. Players may be individual or group of individuals or organizations.

Strategies: Strategies are the actions or the set of actions available to players through which they earn payoffs.

Pay off: Payoffs are the returns to the player at the end of the game. For instance payoffs are the profits in case of profit maximizing firms

Zero sums and non zero sum: A zero sum game is one in which the gain of one player come from the expense of the other player and is exactly equal to the loss of the other player. In other words the sum of two player's payoffs is always zero. Economic applications can be the transaction between a buyer and a seller at the cost price. A non-zero sum game on the other hand is when the gain or loss does not come at the expense of the player. An example of this game might arise if increased advertisement leads to higher profits for various firms or competitors.

Simultaneous vs. Sequential move game: the order of move is significant in game theory. Players in the game may move simultaneously or sequentially which in turn results in different outcomes of the game. A simultaneous move game is a game in which neither player knows the other players actions when moving their actions, that is players take their action simultaneously without knowing the actions that have been chosen by the other players. For instance the cournot model of oligopoly, each firm decides its profit maximizing level of output simultaneously. In the contrast, in

the sequential move game the order of move comes into picture. In this case one player moves first which is then observed by the opponent. The player who moves afterwards gets to observe and learn information about the course of the game up to that point, including what actions other players have chosen. These observations can then be used by that player to decide his/her own optimal strategies, that simply choosing an action. This way strategies of the payers depend on what the other players before have done already.

Representing a Game:

There are two principal representations of the rules of a game

(i) The Normal or strategic form & (ii) the extensive form of rows and columns.

By convention, row corresponds to represent player 1 and column corresponds to player 2. whenever we have a two player game an entry in a cell shows the payoffs of two players for that specific combination strategies chosen. The **extensive** form is the pictorial representation of the rules. The main pictorial form is called the game tree. The game starts from a player 1 from a node and the choice available are represented by branches starting from that node. At the end of a branch player 2 makes his choice and the branches will split into further branches. the game specifies the order in which the player make choices how many times each get to choose and the eventual payoffs to each player for any sequence of choices. **Normal form** is usually used for **simultaneous move games** where as an **extensive** form through a game tree may be used to represents either a simultaneous move or a sequential move game.

III. DOMINATED STRATEGY

Let us consider the well known non co-operative game and a game of complete information, the prisoner’s dilemma. A crime is committed for which there is no eye witness. Two suspects A and B are caught and imprisoned in separate cells. Thus each prisoner’s are in a solitary confinement with no means of communications with another. The magistrate speaks to each of them separately act as an informer. If one of them confesses the crime, he will be freed but the other one will have to spend 4 years in prison. If both of them confess each will have to spend 3 years in prison, but if both of them not confess then the crime cannot be probed and they will be punished for a minimum 1 year’s imprisonment. We can represent it as follows-

	Confess	Not Confess
Confess	(3,3)	(0,4)
Not Confess	(4,0)	(1,1)

When we consider the payoffs associated with different decision option, we realize if one payer chooses to confess then the other player would prefer to confess and be in the position for three years of imprisonment, rather than opting not confess and suffer 4 years of imprisonment. In the same line when one chooses not confess the other would prefer confess so that he get released rather than opting not confess and suffer imprisonment for one year. In both the situations and for both the players we observe the option ‘Not Confess’ is dominated by the option ‘Confess’. i.e. “Not confess” strategy is dominated by the strategy ‘Confess’. Rational players in an attempt to maximize their possible payoffs will not opt for a strictly dominated strategy. On prisoner’s dilemma game rational players will choose to confess, and hence the equilibrium outcome of the game will be (Confess, Confess) resulting the payoff (-3,-3) such a result not optimal when compared with the payoff (-1,-1) associated with strategy profile (Not Confess, Not Confess). In other words both players will suffer only one years of imprisonment when they opt for the strategy “Not Confess”, but they end up opting for the strategy “Confess” and suffer 3 years of imprisonment.

Generally in an n-player game, among strategies S_i^j and $S_i^{j'}$ each belonging to strategy set S_i of an arbitrary player i. strategy S_i^j is dominated by $S_i^{j'}$ if player i’s payoff from plying S_i^j is lower than the payoff he attains from playing $S_i^{j'}$ for each feasible combination of other players’ strategies.

DOMINATED STRATEGY EQUILIBRIUM

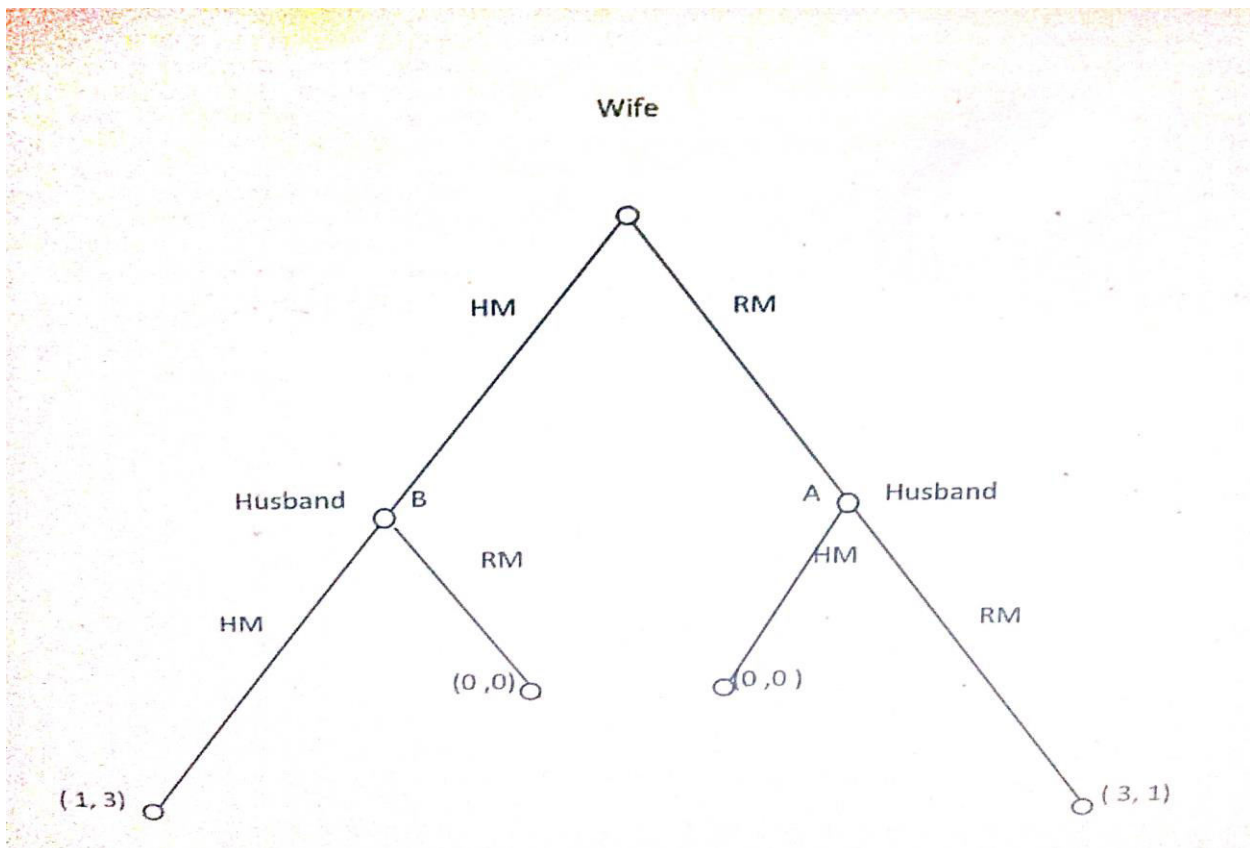
A rational player is expected to play his dominant strategy and thus if all players have a dominant strategy then it is rational for them to choose the dominant strategies and this way we reach at the dominant strategy equilibrium. In terms f the prisoner’s dilemma both the payers have “confess” as dominant strategy, that is in that case each players best response would be to play confess regardless of what the other player might opt for. Thus option (confess, confess) re[representing payoff (-3,-3) as the dominated strategy equilibrium in case of prisoner’s dilemma.

NASH EQUILIBRIUM: Nash equilibrium is a solution concept of game theory involved in two or more players in which each player is assumed to know the equilibrium strategies of each of the other player and no player has anything to gain by changing their strategies unilaterally. The Nobel Laureate John Von Neumann proposed the then concept Nash Equilibrium as a collective set of strategy for each player who takes decision taking into account the other player's decision. However Nash equilibrium does not always mean best collective pay off for all the players involved. It can be applied to various fields such as economics, political science, sociology, psychology etc. In economics it has been widely applied to analyze market competition, pricing strategies, industrial organizations etc.

SEQUENTIAL GAME: In many game however one player can move before the other player, such game are referred to as sequential move game where players take action at well defined turns over time and have perfect knowledge about what the other players did at previous turn. In other words in a two player sequential game one player takes an action before another player. The second mover observes the action taken by first player before it decides what action it should take. As for example if we consider the game battle of sexes where the game involved two players Husband and Wife; planning an evening out with both preferring to go together rather than going alone. Wife wants to enjoy **Romantic Movie (RM)**; While Husband prefers to enjoy **Horror Movie (HM)**. The Normal form of the game can be shown as –

		Husband	
		RM	HM
Wife	RM	(3,1)	(0,0)
	HM	(0,0)	(1,3)

Whereas in sequential move we assume instead of them both moving simultaneously. The wife is the first to move and makes a choice between Romantic Movie and Horror movie, on observing which husband will decides next between the two options.



If we observe the oval around the decision nodes of player 2 (that was there in case of simultaneous-move game) has removed. This is because with sequential moves, Husband (second mover) can observe his wife’s (first mover) action and hence is aware of the decision node he is in on before he takes his action. The wife’s possible strategies remain unchanged at **Romantic movie** or **Horror movie**. But the husband’s set of possible strategies has expanded to four strategies; as for each of the wife’s two actions, he can choose one of the two actions. So the husband has the option to move either from node A or from node B, that is he is not in the same information base. Note that to define husband’s strategies completely we must define an action at each information set that he makes a choice at, even if that information is not actually reached in the game. Thus all strategies in his strategy profile include an action for both the top and bottom information sets. To solve for Nash equilibria, we will consider the normal form of game and use the method of underlying payoffs for best responses.

Always RM (RM/RM) (RM/HM)	Follow wife (RM/RM) (HM/HM)	Opposite to wife (HM/RM) (RM/HM)	always HM (HM/RM) (HM/HM)		
Husband					
Wife	RM	(3,1)	(3,1)	(0,0)	(0,0)
	HM	(0,0)	(1,3)	(0,0)	(1,3)

Here we have 2x4 matrix where wife has a strategy profile $S_w = \{ \text{Romantic movie, Horror movie} \}$ and husband has strategy profile $S_H = \{ (RM, RM), (RM, HM), (RM, RM), (HM, HM), (HM, RM), (RM, HM), (HM, RM), (HM, HM) \}$. Here (RM, RM) and (RM, HM) for instance, means that husband opts for RM given that wife has chosen RM while husband opts for RM after wife has chosen to go for HM. Now the underlining for the best responses ensuring we underline the payoffs for all the strategies that tie for the best responses as well as shown below-

Husband					
Wife	RM	(3,1)	(3,1)	(0,0)	(0,0)
	HM	(0,0)	(1,3)	(0,0)	(1,3)

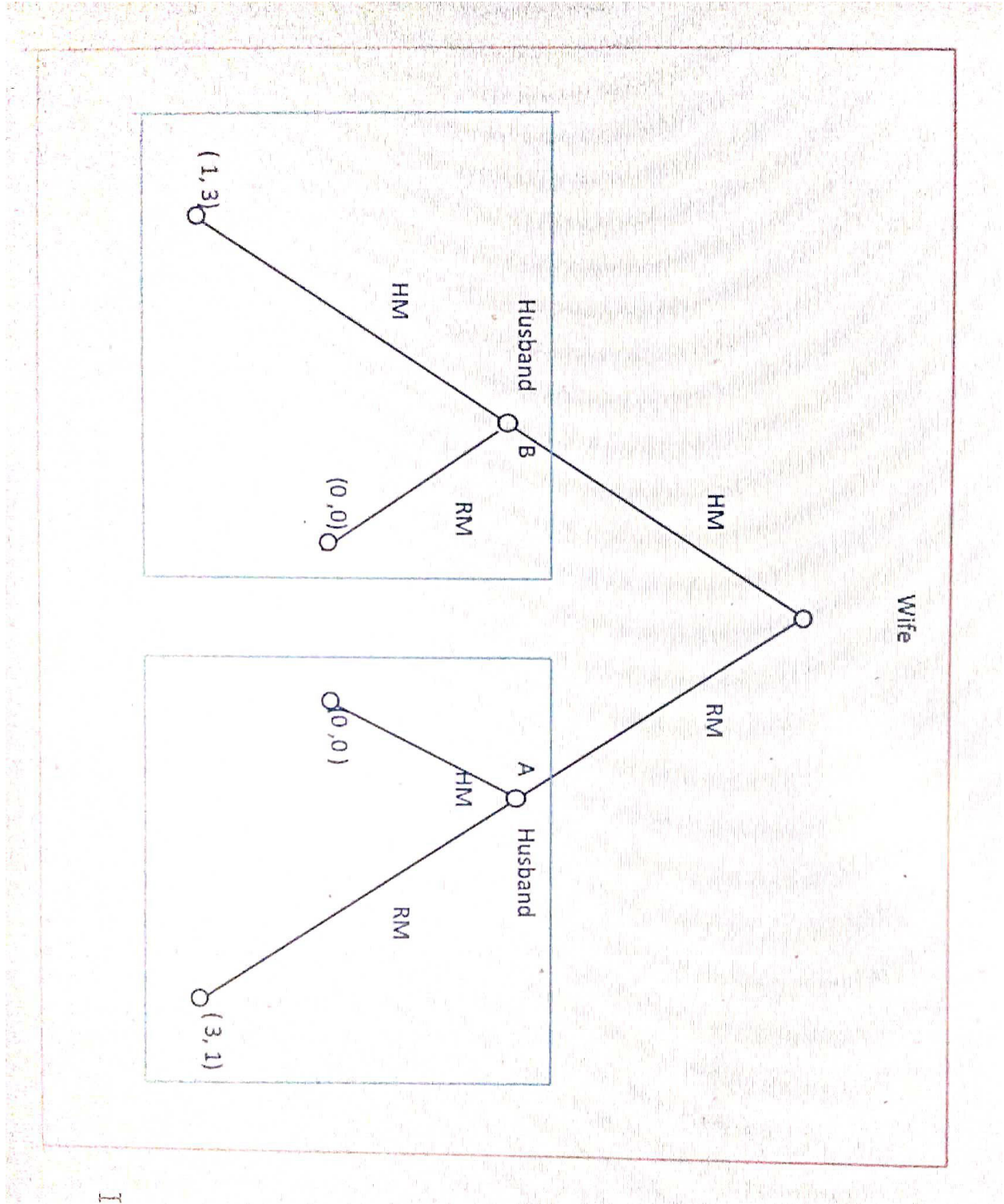
This way we get multiple Nash equilibrium.

1. Wife opts RM husband opts (RM,RM), (RM,HM)
2. Husband Follows his wife’s option (RM,RM), (HM,HM)
3. Wife opts HM husband opts (HM,RM), (HM,HM)

Among the three equilibria, let us analyze the probability of each we start with the first Nash equilibrium husband’s strategy (RM, RM), (RM, HM) involves that a threat to wife that he will choose RM when his wife will go for HM. However this is an empty threat. This results from the fact that if the wife really were to choose a HM first then husband would be giving up a payoff of 3 by choosing RM rather than HM, which is certainly hard to believe. Thus husband’s threat to his wife to opt for RM if the wife goes for a HM is non-credible. On the similar lines of husband’s strategy in the third Nash equilibrium (HM, RM), (HM, HM) involves an empty threat of him choosing to go for HM when his wife opts for RM in the first move. As for this he will be giving up a payoff of 1 by choosing HM rather than RM.

IV. SUB GAME PERFECT NASH EQUILIBRIUM

Nash equilibrium in sequential game may result in strategy profiles which are not very unlikely. A formal procedure for selecting a reasonable Nash equilibrium strategy profile is done using the concept of sub game perfect Nash equilibrium which requires that for equilibrium to be rational it should be equilibrium not just for the game as a whole, but also for each sub game of the game. A sub game is a part of the extensive form beginning from any node of the game and including everything that branches out below it. A decision node initiates a sub game if neither it nor any of its successors are in the same information set that contains nodes that are not successors to it. Such a sub game is termed as proper sub game this way the sequential Battle of sexes game has three sub games. Viz. the game itself and the two lower sub games beginning from the nodes where the husband gets time to decide. We can show these as following figure-



A sub perfect Nash equilibrium is a set of strategies one for each player that induce a Nash equilibrium in every proper sub game. if we concern the case of sequential battle of sexes game, for a strategy profile to be termed as a SPNE, in addition to it being a Nash equilibrium on the whole game, it must be the Nash equilibrium of the two other proper sub game. In the above set-up, when wife chooses RM, in the sub game (ii) then Husband has to decide between RM (that results payoff of 1) and HM (that gives payoff f 0). His best response in this sub game will be to go for RM. Similarly, when wife chooses to go for HM, husband in the sub game (iii) is to decide between RM (giving payoff 0) and HM (giving payoff 3). His best response would be to go for a Horror Movie (HM). Thus, (RM, RM), (HM, HM) is the only strategy profile of the husband that forms a part of the SPNE. The other two strategy profile like (RM, RM), (RM, HM) and (HM, RM), (HM, HM) results in him playing something that is not a Nash equilibrium on some proper sub game. Thus among the three Nash equilibrium we come across for the game above only the second one is a sub game perfect Nash equilibrium while the first and the third are not.

APPLICATION OF NASH EQUILIBRIUM IN SEQUENTIAL MOVE

Example (1). Let us consider two symmetric firms A and B having same constant average cost Rs 2 per unit. The firm faces a total market demand of 100 and 180 units corresponding to two different prices. i.e. either a high price f Rs 10 or at a low price Rs 5 respectively. In both the cases the market demand is splitted between the two firms. If one firm sets high price and the other low price, the low price firms sells 150 units and high priced firm sells only 20 units of the commodity. Now we can calculate the profile of these firms and derive the equilibrium set of strategies using prisoner’s dilemma game. We have the following possible strategy profile for (firm A, firm B) and their associated payoffs as (firm A, firm B)

1. (High price, High price): in this case total demand=100, so each firm will sell 50. Total revenue of an individual firm=50x10= Rs500 & total cost=Rs100
2. Therefore profit = Rs 500- Rs 100= Rs 400 for each firm.
3. (Low price, Low price): Here total demand=180, so each firm will sell 90. Total revenue of an individual firm=90x5=450 & total cost=Rs 180, so profit Rs 450- Rs180= Rs 270 for each firm
4. (High price, Low price): here suppose firm A sells at high price, its total demand is 20 units so Revenue from firm A =20x10=200 and Total cost =20x2=Rs 40., thereby profit=Rs 200-Rs 40 = Rs 160 for firm A. Firm B sell at Low price, its demand is 150, so revenue=150x5=Rs 750 and Total cost= 150x2=Rs 300, thereby profit= R750-300= Rs 450 for firm B .
5. (Low price, High price): contrary to the above point, assume firm A sells at low price and firm B sells at high price. After calculation we find firm A profits Rs 450 and firm B profit Rs 160.

The payoff matrix therefore can be written as

		B's Payoff	
		High price	Low price
A's Payoff	High price	(400,400)	(160,450)
	Low price	(450,160)	(270,270)

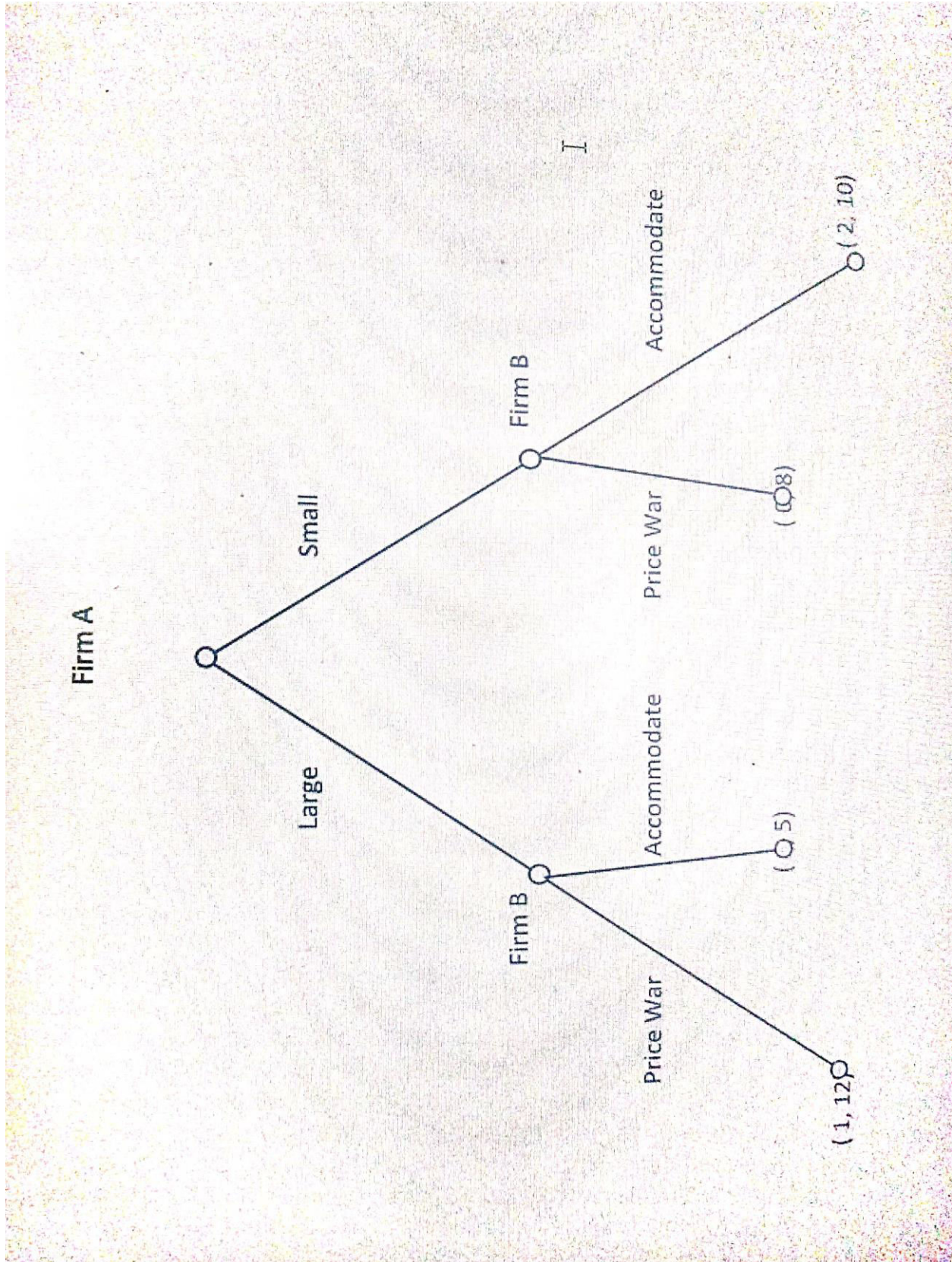
Price the equilibrium is therefore (Low price, Low price) yielding payoff (270,270). It is a scenario similar to Prisoner’s dilemma game where the equilibrium outcome is the one which gives lowest joint payoff (540). The strategy (Low price, Low price) is the Nash equilibrium as well as dominant strategy equilibrium. However the outcome (HP, HP) with a highest joint payoff (800) is a Pareto improvement over Nash equilibrium outcome. A question arises why this paradoxical situation arises in equilibrium. From the dominant strategy equilibrium perspective obviously none f the firm will choose the dominated strategy (HP) over the dominant strategy. Hence the dominated strategy equilibrium is (LP, LP). Let us now analyze it from the perspective of Nash equilibrium. From the payoff it is evident that even though each of the firm has an incentive to choose High price strategy because the strategy (HP, HP) offers highest joint profit, but each firm knows that if it chooses (HP) the rival firm has incentive to deviate and its best response is to choose (LP) and appropriate higher profit of Rs 450 (450>400). Thus there is a mutual threat for deviation for lower

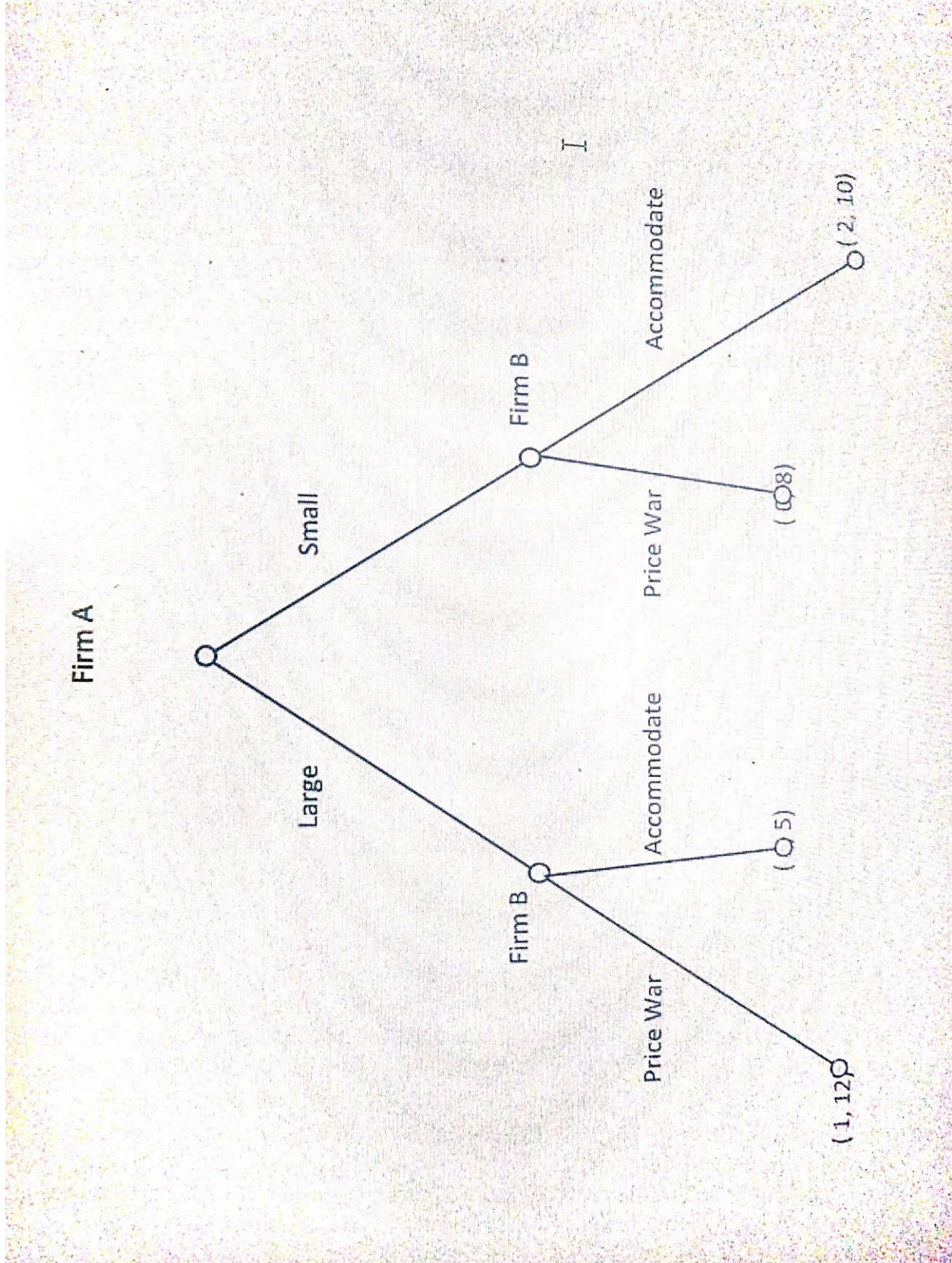
price against the choice of High price. This credible threat actually compels the firm to choose the mutually best response strategy, even though it is a sub optimal outcome (LP, LP).

Example (2). Let there be a firm A which is trying to enter a business, either on a small scale or large scale. Firm B now has to decide whether to accommodate or start a price war. So the game essentially involves the two strategies of whether a firm enters business on a large scale or small scale. Consider a pay matrix of the form below –

		Firm- B	
		Accommodate	Price war
Firm-A	small	(2,10)	(0,8)
	Large	(4,5)	(<u>1</u> , <u>12</u>)

As per above payoff matrix , for firm A “large” is the dominated strategy, given firm A choose this, firm B will respond by launching a “price war“. At this stage Nash equilibrium will be (Large, Price war) where firm A will have a payoff 1 and firm B a payoff 12. But firm A can do better if we turn it into a sequential move game. Here firm A can commit to a scale of operation in advance. If firm A decide “large”, then firm B’s best response is to fight a “price war” resulting in firm A’s payoff to be 1, but if firm A choose “small” then firm Bs best response will be to accommodate yielding payoff of 2 to firm A. hence sub game perfect Nash equilibrium strategy in this sequential move game is (Small, Accommodate, Large, price war) where firm A enters “ small” and firm B “accommodate” in the top information set, and engages in a price war in the bottom information set hence,





equilibrium payoff is (2, 10). The sub game perfect Nash equilibrium outcome is small, accommodate). This shows moving first can have strategic value.

V. CONCLUSION

In this paper I have introduced the concept of game theory, the dominated strategies and Nash equilibrium for solving a game especially in sequentially move game. Besides I have given two examples as their applications in various fields. Although Nash equilibrium is a very big concept I have tried to give a brief idea of it in this paper. In this paper I have tried to explore the applications of Nash equilibrium in different fields, and look forward to the prospects in the field of machine learning and artificial intelligence.

Nash equilibrium is an important concept in game theory that can be applied to various fields such as Economics, political science, sociology, psychology etc. In economics it has been widely applied to analyze market competition, pricing strategies, industrial organization, etc. thus the concept and methods of Nash equilibrium are of great importance for us to better understand human behavior and social phenomena.

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