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(S, d) Magic Labeling of Splitting Graphs

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ABSTRACT: Let $G(p, q)$ be a connected, undirected, simple and non-trivial graph with p vertices and q edges. Let f be an injective function $f: V(G) \rightarrow \{s, s+d, s+2d, \dots, s+(q+1)d\}$ and g be an injective function $g: E(G) \rightarrow \{d, 2d, 3d, \dots, 2(q-1)d\}$. Then the function f is said to be (s, d) magic labeling if $f(u) + g(uv) + f(v)$ is a constant, for all $u, v \in V(G)$ and $uv \in E(G)$. A graph G is called (s, d) magic graph if it admits (s, d) magic labeling.

KEYWORDS: $\text{spl}(P_n)$, $\text{spl}(P_n \odot K_1)$, $\text{spl}(K_{1,n})$, $\text{spl}(B_{m,n})$

I. INTRODUCTION

[1] Graph labeling involves assigning labels to the vertices, edges, or both in a graph, following certain specified conditions. One specific type of labeling is edge-magic labeling, where a bijection is created from the union of the vertex set V and the edge set E to the set $\{1, 2, \dots, p+q\}$, where p is the number of vertices and q is the number of edges. The labeling must satisfy the condition that for each edge $(u, v) \in E$, the sum $f(u) + f(u, v) + f(v) = s$, where s is a constant for all edges.

Edge-magic labeling was introduced by Neelam Kumari and Seema Mehra in 2013. In this paper the existence of (s, d) magic labeling of $\text{spl}(P_n)$, $\text{spl}(P_n \odot K_1)$, $\text{spl}(K_{1,n})$ and $\text{spl}(B_{m,n})$

[5] Let $G(p, q)$ be a simple, non-trivial, connected, undirected graph with p vertices and q edges. Consider the following: $f: V(G) \rightarrow \{s, s+d, s+2d, \dots, s+(q+1)d\}$ and $g: E(G) \rightarrow \{d, 2d, 3d, \dots, 2(q-1)d\}$ be an injective function. Then, for any $u, v \in V(G)$ and $uv \in E(G)$, $f(u) + g(uv) + f(v)$ is a constant, and the function f is said to be (s, d) magic labeling. If a graph G admits (s, d) magic labeling, then it is referred to as a (s, d) magic graph.

II. DEFINITIONS

Definition 2.1:

For a graph G the split graph which is denoted as $\text{spl}(G)$ is obtained by adding to each vertex v a new vertex v' such that v' is adjacent to every vertex that is adjacent to v in G .

Definition 2.2:

A path can be denoted as a sequence of vertices $P = v_1, v_2, \dots, v_k$ where v_1, v_2, \dots, v_k are the vertices in the graph and for every i , there is an edge between v_i and v_{i+1}

Definition 2.3:

$P_n \odot K_1$ represents the corona product of two graphs: a path graph P_n and a complete graph K_1 .

Definition 2.4:

A star graph, denoted as $K_{1,n}$, is a graph consisting of one central vertex connected to n outer vertices

Definition 2.5:

The graph bistar $B_{m,n}$ is the graph obtained by joining the centre vertices of two copies of $K_{1,n}$ with an edge of $B_{m,n}$.

III. MAIN RESULT

Theorem 3.1: The splitting of a path graph is (s, d) magic labeling

Proof: Let $G = spl(P_n)$ has $2n$ vertices and $3n - 3$ edges.

Let $V(spl(P_n)) = \{v_i : 1 \leq i \leq \eta\} \cup \{u_i : 1 \leq i \leq \eta\}$ and

$E(spl(P_n)) = \{(u_i v_{i+1}) : 1 \leq i \leq \eta - 1\} \cup \{(v_i v_{i+1}) : 1 \leq i \leq \eta - 1\} \cup \{(u_i u_{i+1}) : 2 \leq i \leq \eta - 1\}$

Define the function f from the vertex set to $\{s, s + d, s + 2d \dots s + (q + 1)d\}$,

$g: E(G) \rightarrow \{d, 2d, 3d \dots 2(q - 1)d\}$

Case (a) when η is odd

Labeling of vertices				
Value of i	$f(v_{2(i+1)})$	$f(v_{2i+1})$	$f(u_{2(i+1)})$	$f(u_{2i+1})$
$0 \leq i \leq \frac{\eta - 3}{2}$	$s + (2(1 + 2i)d)$	–	$s + (3 + 4i)d$	–
$0 \leq i \leq \frac{\eta - 1}{2}$	–	$s + (1 + 4i)d$	–	$s + 4id$

Table 1 Labeling of vertices of $spl(P_n)$

Labeling of Edges			
Value of i	$g(v_i v_{i+1})$	$g(u_i v_{i+1})$	$g(v_i u_{i+1})$
$1 \leq i \leq \eta - 1$	$2s + 2(q - 1)d - (f(v_i) + f(v_{i+1}))$	$2(q - 1)d - (f(u_i) + f(v_{i+1}))$	$2s + 2(q - 1)d - (f(v_i) + f(u_{i+1}))$

Table 2 Labeling of Edges of $spl(P_n)$

Case (b) when η is even

Labeling of vertices				
Value of i	$f(v_{2(i+1)})$	$f(v_{2i+1})$	$f(u_{2(i+1)})$	$f(u_{2i+1})$
$0 \leq i \leq \frac{\eta}{2} - 1$	$s + (2(1 + 2i)d)$	$s + (1 + 4i)d$	$s + (3 + 4i)d$	$s + 4id$

Table 3 Labeling of vertices of $spl(P_n)$

Labeling of Edges			
Value of i	$g(v_i v_{i+1})$	$g(u_i v_{i+1})$	$g(v_i u_{i+1})$
$1 \leq i \leq \eta - 1$	$2s + 2(q - 1)d - (f(v_i) + f(v_{i+1}))$	$2s + 2(q - 1)d - (f(u_i) + f(v_{i+1}))$	$2(q - 1)d - (f(v_i) + f(u_{i+1}))$

Table 4 Labeling of Edges of $spl(P_n)$

Therefore $f(v_i) + f(v_{i+1}) + g(v_i v_{i+1}), f(u_i) + f(u_{i+1}) + g(u_i u_{i+1}), f(v_i) + f(u_{i+1}) + g(v_i u_{i+1})$, are constant equals to $2(s + (q - 1)d)$. Hence $spl(P_n)$ admits (s, d) magic labeling.

Example 3.1 splitting of a path graph $spl(P_7)$

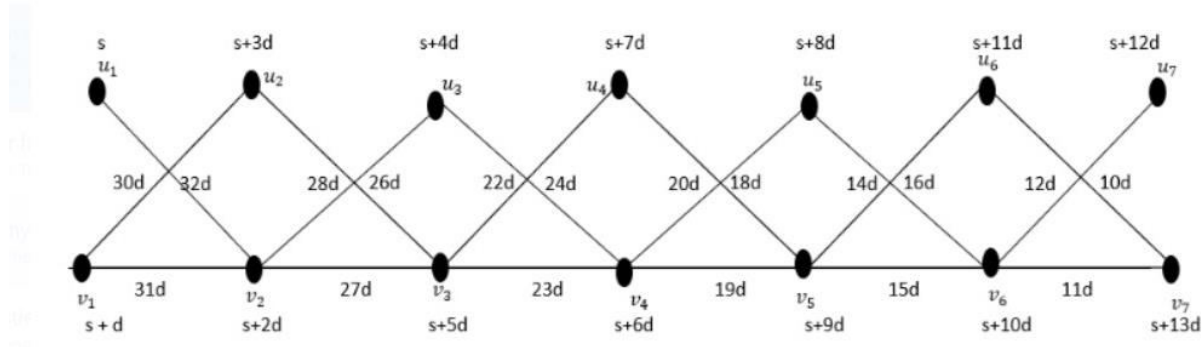


Fig. 1 splitting of a path graph $spl(P_7)$

Theorem 3.2: The splitting of comb graph $spl(P_n \odot K_1)$ is (s, d) magic labeling.

Proof:

Let $V(P_n \odot K_1) = \{v_i, v'_i \cup u_i, u'_i; 1 \leq i \leq \eta\}$ and

$E(P_n \odot K_1) = (v_i v_{i+1}), (u_i u_{i+1}), (u_{i+1} v_i); 1 \leq i \leq \eta - 1 \cup \{(v'_i v_i), (u'_i v_i), (v'_i u_i); 1 \leq i \leq \eta\}$.

Here $|V(P_n \odot K_1)| = 4\eta$ and $|E(P_n \odot K_1)| = 6\eta$. Define the function f from the vertex set to $\{s, s + d, s + 2d \dots s + (q + 1)d\}$, $g: E(G) \rightarrow \{d, 2d, 3d \dots 2(q - 1)d\}$

Case (a) When η is odd

Labeling of vertices								
Value i	$f(u_{2i+1})$	$f(u_{2(i+1)})$	$f(v_{2i+1})$	$f(v_{2(i+1)})$	$v'_{(v_{2i+1})}$	$v'_{(2(i+1))}$	$u'_{(2(i+1))}$	$u'_{(2i+1)}$
$0 \leq i \leq \frac{\eta - 1}{2}$	$S + 8id$	—	$s + (8i + 1)d$	—	$s + (8i + 2)d$	—	$s + (8i + 4)d$	—
$0 \leq i \leq \frac{\eta - 3}{2}$	—	$s + (8i + 7)d$	—	$s + (8i + 6)d$	—	$s + (8i + 5)d$	—	$s + (8i + 3)d$

Table 5: Labeling of vertices of the graph $spl(P_n \odot K_1)$

Labeling of Edges						
Value of i	$g(v_i v_{i+1})$	$g(u_i u_{i+1})$	$g(v_i u_{i+1})$	$g(v_i v'_i)$	$g(v_i u'_i)$	$g(u_i v'_i)$
$1 \leq i \leq \eta - 1$	$2s + 2(q - 1)d - (f(v_i) + f(v_{i+1}))$	$2s + 2(q - 1)d - (f(u_i) + f(u_{i+1}))$	$2s + 2(q - 1)d - (f(v_i) + f(u_{i+1}))$	—	—	—

$1 \leq i \leq \eta$	–	–	–	$2s + 2(q - 1)d - (f(v_i) + f(v_i'))$	$2s + 2(q - 1)d - (f(v_i) + f(u_i'))$	$2s + 2(q - 1)d - (f(u_i) + f(v_i'))$
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Table 6: Labeling of edges of the graph $spl(P_n \odot K_1)$

Case (b) when η is even

Labeling of vertices								
Value i	$f(u_{2i+1})$	$f(u_{2(i+1)})$	$f(v_{2i+1})$	$f(v_{2(i+1)})$	$v'_{(2i+1)}$	$v'_{(2(i+1))}$	$u'_{(2(i+1))}$	$u'_{(2i+1)}$
$0 \leq i \leq \frac{\eta}{2} - 1$	$s + 8id$	$s + (8i + 7)d$	$s + (8i + 1)d$	$s + (8i + 6)d$	$s + (8i + 2)d$	$s + (8i + 5)d$	$s + (8i + 4)d$	$s + (8i + 3)d$

Table 7: Labeling of vertices of the graph $spl(P_n \odot K_1)$

Labeling of Edges						
Value of i	$g(v_i v_{i+1})$	$g(u_i u_{i+1})$	$g(v_i u_{i+1})$	$g(v_i v_i')$	$g(v_i u_i')$	$g(u_i v_i')$
$1 \leq i \leq \eta - 1$	$2s + 2(q - 1)d - (f(v_i) + f(v_{i+1}))$	$2s + 2(q - 1)d - (f(u_i) + f(u_{i+1}))$	$2s + 2(q - 1)d - (f(v_i) + f(u_{i+1}))$	–	–	–
$1 \leq i \leq \eta$	–	–	–	$2s + 2(q - 1)d - (f(v_i) + f(v_i'))$	$2s + 2(q - 1)d - (f(v_i) + f(u_i'))$	$2s + 2(q - 1)d - (f(u_i) + f(v_i'))$

Table 8: Labeling of edges of the graph $spl(P_n \odot K_1)$

Therefore $f(v_i) + f(v_{i+1}) + g(v_i v_{i+1}), f(u_i) + f(u_{i+1}) + g(u_i u_{i+1}), f(v_i) + f(u_{i+1}) + g(v_i u_{i+1}), f(v_i') + f(v_i) + g(v_i v_i'), f(v_i) + f(u_i') + g(v_i u_i'), f(u_i) + f(v_i') + g(u_i v_i')$ are constant equals to $2(s + (q - 1)d)$. Hence the $spl(P_n \odot K_1)$ admits (s, d) magic labeling.

Example 3.2 splitting of comb graph spl (P₅ ⊙ K₁)

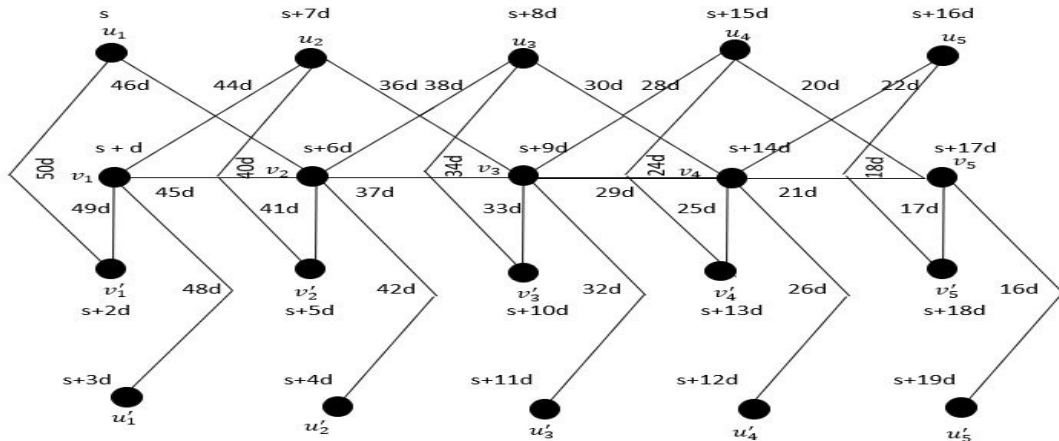


Fig.2 splitting of comb graph spl (P₅ ⊙ K₁)

Theorem 3.3: The graph Spl (B_{m,η}) m, η ≥ 2 is (s, d) magic labeling.

Proof: Let G = Spl (B_{m,η}) has 2m + 2η + 4 vertices and 3m + 3η + 3 edges

Let the vertex set be V (spl(B_{m,η})) = {v_i, v'_i : 1 ≤ i ≤ η + 1} ∪ {u_i, u'_i : 1 ≤ i ≤ m + 1},

E(Spl(B_{m,η})) = {(u₁u_{i+1}), (u₁u'_{i+1}), (u'₁u'_{i+1}), : 1 ≤ i ≤ m} ∪ {(v'₁v_{i+1}), (v₁v_{i+1}), (v₁v'_{i+1}): 1 ≤ i ≤ η} ∪ {(u₁v'₁), (u₁v₁), (u'₁v₁)}

Define the function f from the vertex set to {s, s + d, s + 2d ... s + (q + 1)}, g: E(G) → {d, 2d, 3d ... 2(q - 1)d}

Labeling of vertices				
f(u ₁) = s, f(v ₁) = s+d				
f(v' ₁) = f(v' _{η+1}) + 3d				
f(u' ₁) = f(v' _{η+1}) + d				
Value of i	f(u _{i+1})	f(u' _{i+1})	f(v _{i+1})	f(v' _{i+1})
1 ≤ i ≤ η	-	-	f(u' _{m+1}) + id	f(v' _{η+1}) + id
1 ≤ i ≤ m	s + (i + 1)d	f(u _η) + id	-	-

Table 9: Labeling of vertices of the graph Spl(B_{m,η})

Labeling of edges						
g((u ₁ v' ₁) = 2s + 2(q - 1)d - (f(u ₁) + f(v' ₁))						
g(u ₁ v ₁) = 2s + 2(q - 1)d - (f(u ₁) + f(v ₁))						
g(u' ₁ v ₁) = 2s + 2(q - 1)d - (f(u' ₁) + f(v ₁))						
Value i	g(u ₁ u _{i+1})	g(u ₁ u' _{i+1})	g(u' ₁ u' _{i+1})	g(v' ₁ v _{i+1})	g(v ₁ v _{i+1})	g(v ₁ v' _{i+1})
1 ≤ i ≤ m	2s + 2(q - 1)d - (f(u ₁) + f(u _{i+1}))	2s + 2(q - 1)d - (f(u ₁) + f(u' _{i+1}))	2s + 2(q - 1)d - (f(u' ₁) + f(u' _{i+1}))	-	-	-

$1 \leq i \leq \eta$	–	–	–	$2s + 2(q - 1)d - (f(v'_1) + f(v_{i+1})) + f(v_{i+1})$	$2s + 2(q - 1)d - (f(v_1) + f(v_{i+1})) + f(v_{i+1})$	$2s + 2(q - 1)d - (f(v_1) + f(v'_{i+1})) + f(v'_{i+1})$
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Table 10: Labeling of edges of the graph $Spl(B_{m,\eta})$

Therefore $f(u_1) + f(v'_1) + g(u_1v'_1)$, $f(u_1) + f(v_1) + g(u_1v_1)$, $f(u'_1) + f(v_1) + g(v_1u'_1)$, $f(u_1) + f(u_{i+1}) + g(u_1u_{i+1})$, $f(u_1) + f(u'_1) + g(u_1u'_{i+1})$, $f(u'_1) + f(u'_{i+1}) + g(u'_1u'_{i+1})$, $f(v'_1) + f(v_{i+1}) + g(v'_1v_{i+1})$, $(f(v_1) + f(v_{i+1})) + g(v_1v_{i+1})$, $f(v_1) + f(v'_{i+1}) + g(v_1v'_{i+1})$ are constant equals to $2(s + (q - 1)d)$. Hence the $Spl(B_{m,\eta})$ admits (s, d) magic labeling.

Example 3.3 $Spl(B_{3,4})$

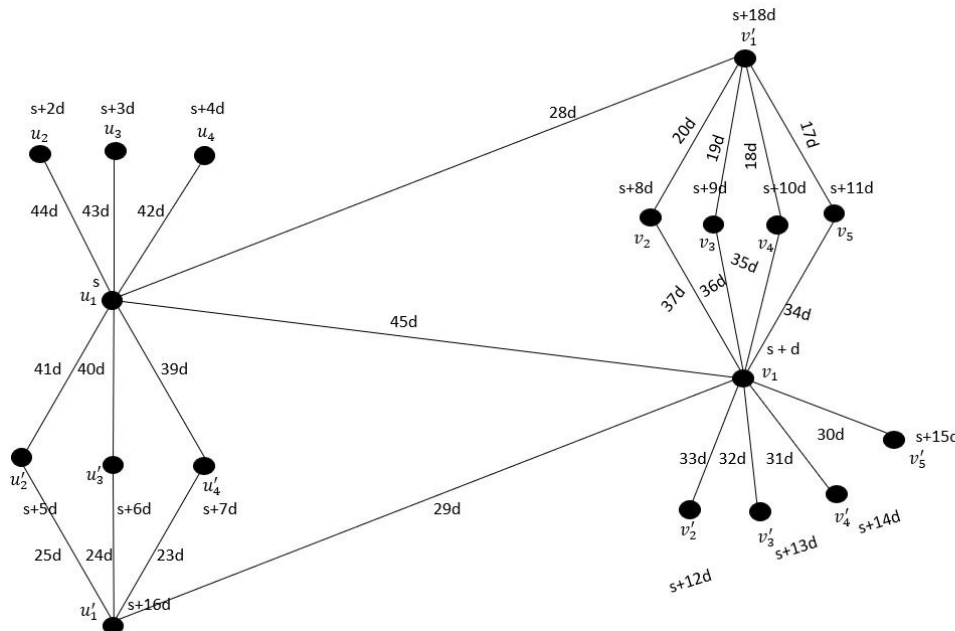


Fig.3 $Spl(B_{3,4})$

Theorem 3.4: The star graph $S(K_{1,\eta})$ is (s, d) magic labelling.

Proof: Let $G = Spl(K_{1,\eta})$. Let $x_1, v_1, v_2, \dots, v_\eta$ are the vertices of the graph $K_{1,\eta}$. Then $x'_1, v'_1, v'_2, \dots, v'_\eta, x_1, v_1, v_2, \dots, v_\eta$ are the vertices of $S(K_{1,\eta})$. Here $|V(S(K_{1,\eta}))| = 2\eta + 2$ and $|E(S(K_{1,\eta}))| = 3\eta$

Define the function f from the vertex set to $\{s, s + d, s + 2d, \dots, s + (q + 1)d\}$, $g: E(G) \rightarrow \{d, 2d, 3d, \dots, 2(q - 1)d\}$

Labeling of vertices		
$f(x_1) = s$		
$f(x'_1) = s + 2(\eta + 1) - 1)d$		
Value i	$f(v_i)$	$f(v'_i)$
$1 \leq i \leq \eta$	$s + id$	$s + (\eta + i)d$

Table 11: Labeling of vertices of the graph $S(K_{1,\eta})$

Labeling of edges			
Value i	$g(x_1v_i)$	$g(x_1v'_i)$	$g(x'_1v'_i)$
$1 \leq i \leq \eta$	$2s + 2(q - 1)d - (f(x_1) + f(v_i))$	$2s + 2(q - 1)d - (f(v'_i) + f(x_1))$	$2s + 2(q - 1)d - (f(x'_1) + f(v'_i))$

Table 12: Labeling of edges of the graph $S(K_{1,\eta})$

Therefore $f(v_i) + f(x_1) + g(v_ix_1)$, $f(v'_i) + f(x_1) + g(x_1v'_i)$ and $f(x'_1) + f(v'_i) + g(x'_1v'_i)$ are constant equals to $2(s + (q - 1)d)$. Hence the $S(K_{1,\eta})$ admits (s, d) magic labeling.

Example 3.4 Star graph $S(K_{1,3})$

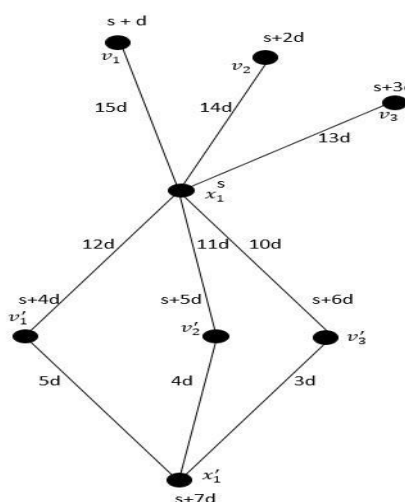


Fig.4 Star graph $S(K_{1,3})$

IV. CONCLUSION

In this paper, we have explored certain types of graphs splitting that permit (s, d) Magic Labeling. Further research could focus on identifying the specific conditions under which various graphs admit (s, d) Magic Labeling. Alternatively, future research could focus on identifying the specific factors or conditions that allow a graph to have this type of labeling.

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